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A STUDY ON ROUGH FUZZY PRIME BI-IDEALS & PRIME FUZZY BI-IDEALS IN SEMIRINGS

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Abstract: In this paper we initiate the study of rough fuzzy prime bi-ideals in semirings. We also define rough prime fuzzy bi-ideals, rough semiprime fuzzy bi-ideals, rough irreducible fuzzy bi-ideals and Strongly rough irreducible fuzzy bi-ideals of semirings .

Index terms: Rough fuzzy prime bi-ideals, rough prime fuzzy bi-ideals, rough semiprime fuzzy bi-ideals, rough irreducible fuzzy bi-ideals, strongly, rough irreducible fuzzy bi-ideals.

1.INTRODUCTION AND HISTORICAL NOTES

L.A.Zadeh[22] introduced the notion of a fuzzy sets, and it is now a rigorous area of research with manifold applications ranging from engineering and computer science to medical diagnosis and social behaviour studies. Rosenfeld[15], applied the notion of fuzzy sets to algebra and introduced the notion of fuzzy subgroups. Liu[10] defined and studied fuzzy subrings and fuzzy ideals of a ring.

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The concept of semiring was introduced by H.S.Vandiver[21]. T.K.Dutta and B.K.Biswas [8] studied fuzzy ideals and fuzzy prime ideals in semirings. S. Bashir [1] introduced prime fuzzy bi-ideals in semirings.

The concept of rough set was introduced by Pawlak[11-14] in 1982. It is a mathematical framework for dealing with uncertainty and so some extend overlapping fuzzy set theory. The rough set theory approach is based on indiscernibility relations and approximations. The theory of rough set is an extension of set theory. So many others contributed different articles on these concepts and applied it on different branches of pure and applied mathematics. R.Chinram[3-5] introduced rough prime ideals and rough fuzzy prime ideals in Γ-semigroups.. N.Thillaigovindan and V.S.Subha [20] studied rough prime bi-ideals in Γ-semigroups. The notion of rough fuzzy set was introduced by Dubois and Parade[7]. G.Senthilkumar and V.selvan[16] studied the lower and upper approximation of fuzzy ideals in semirings. Kanchi and Davvaz[9] introduced rough prime(primary) ideals and rough fuzzy prime and primary ideals in semirings. The notion rough k-ideals introduced by V.S.Subha[17-19] in semiring .

This paper concern the relationship between rough sets and fuzzy sets in semirings. We discuss the lower and upper approximation of fuzzy prime bi-ideals in semirings.

2.PRELIMINARIES

In this section we present some of the preliminaries required for the development. Throughout this paper R denotes the semiring unless otherwise specified.

A fuzzy subset μ of R is non empty if μ is not the constant map which assumes the value 0. For any fuzzy subsets μ and λ of R, $\mu \subseteq \lambda$ means that for all $a \in R$, $\mu(a) \leq \lambda(a)$.

Definition 2.1.[1]Let μ be a fuzzy subset of R. Let $\overline{\rho}(\mu)$ and $\underline{\rho}(\mu)$ be the fuzzy subsets of R defined by $\overline{\rho}(\mu)(x) = \bigvee_{a\in[x]_\rho} \mu(a)$ and $\underline{\rho}(\mu) = \bigwedge_{a\in[x]_\rho} \mu(a)$ are called, respectively, the *ρ-upper* and *ρ-lower approximations* of the fuzzy set μ. $\rho(\mu) = (\underline{\rho}(\mu), \overline{\rho}(\mu))$ is called a *rough fuzzy set* with respect to ρ if $\underline{\rho}(\mu) \neq \overline{\rho}(\mu)$.

Definition 2.2.[1] A fuzzy subset μ of R is called a *fuzzy subsemiring* of R if

i) $\mu(x+y) \geq \mu(x) \wedge \mu(y)$ and

ii) $\mu(xy) \geq \mu(x) \wedge \mu(y)$ for all $x, y \in R$.

Definition 2.3.[1] A fuzzy subset μ of R is called a *fuzzy left(right) ideal* of R if

i) $\mu(x+y) \geq \mu(x) \wedge \mu(y)$ and

ii) $\mu(xy) \geq \mu(x)(\mu(xy) \geq \mu(y))$ for all $x, y \in R$.

If μ is both a fuzzy left ideal and fuzzy right ideal of R then it is called a fuzzy ideal of R.

Definition 2.4.[1] A fuzzy subset μ of R is called a *fuzzy bi-ideal* of R if

i) $\mu(x+y) \geq \mu(x) \wedge \mu(y)$

ii) $\mu(xy) \geq \mu(x)(\mu(xy) \geq \mu(y))$ and

iii) $\mu(xzy) \geq \mu(x) \wedge \mu(y)$ for all $x, y, z \in R$.

Definition 2.5[1].Let μ be a fuzzy subset of $R, t \in [0,1]$. Then the sets $\mu_t = \{x \in R \mid \mu(x) \geq t\}$ and $\mu_t^R = \{x \in R \mid \mu(x) > t\}$ are called respectively t-*level set* and t-*strong level set* of the fuzzy set μ.

Theorem 2.6[1]. *Let μ be a fuzzy subset of R. Then μ is a fuzzy bi-ideal of R if and only if $\mu_t \neq \emptyset$ be a bi-ideal of R for every $t \in [0,1]$.*

Theorem 2.7[1]. *Let μ be a fuzzy subset of R. Then μ is a fuzzy bi-ideal of R if and only if for all $t \in [0,1]$, if $\mu_t^R \neq \emptyset$ then μ_t^R be a bi-ideal of R.*

Definition 2.8.[1] A fuzzy bi-ideal μ of a semiring R is called a *fuzzy prime bi-ideal* of R if $\mu(xzy) = \mu(x)$ or $\mu(xzy) = \mu(y)$.

Definition 2.9.[1] A fuzzy bi-ideal μ of R is called a *prime fuzzy bi-ideal* of R if for any bi-ideals f, g of R, $f \circ g \leq \mu$ implies $f \leq \mu$ or $g \leq \mu$.

A fuzzy bi-ideal μ of R is called a *strongly prime fuzzy bi-ideal* of R if for any bi-ideals f, g of R, $f \circ g \wedge g \circ f \leq \mu$ implies $f \leq \mu$ or $g \leq \mu$.

A fuzzy bi-ideal λ of R is said to be *idempotent* if $\lambda = \lambda \circ \lambda = \lambda^2$.

A fuzzy bi-ideal μ of R is called a *semiprime fuzzy bi-ideal* of R if for any bi-ideals f, g of R, $g \circ g = g^2 \leq \mu$ implies $g \leq \mu$ for every fuzzy bi-ideal g of R.

Definition 2.10.[1] A fuzzy bi-ideal μ of R is said to be an *irreducible fuzzy bi-ideal* of R if for any bi-ideals f and g of R, $f \wedge g = \mu$ implies either $f = \mu$ or $g = \mu$.

A fuzzy bi-ideal μ of R is said to be an *Strongly irreducible fuzzy bi-ideal* of R if for any bi-ideals f and g of R, $f \wedge g \leq \mu$ implies either $f \leq \mu$ or $g \leq \mu$.

3. ROUGH FUZZY PRIME BI-IDEALS IN SEMIRINGS

In this section we introduce the notion of rough fuzzy prime bi-ideal of a semiring R and study some of its properties.

Theorem 3.1. *Let ρ be a congruence relation on R and let μ be a fuzzy subset of R. If μ is a fuzzy subsemiring of R, then $\overline{\rho}(\mu)$ is a fuzzy subsemiring of R.*

Proof: Let μ be a fuzzy subsemiring of R. Let $x, y \in R$. Then $\mu(x+y) \geq \mu(x) \wedge \mu(y)$ and $\mu(xy) \geq \mu(x) \wedge \mu(y)$. We have

(i)
$$\begin{aligned}\overline{\rho}(\mu)(x+y) &= \vee_{r \in [x+y]_\rho} \mu(r) \\ &= \vee_{r \in [x]_\rho + [y]_\rho} \mu(r) \\ &= \vee_{l \in [x]_\rho,\ m \in [y]_\rho} \mu(lm) \\ &\geq \left(\vee_{l \in [x]_\rho} \mu(l)\right) \wedge \left(\vee_{m \in [y]_\rho} \mu(m)\right) \\ &= \overline{\rho}(\mu)(x) \wedge \overline{\rho}(\mu)(y)\end{aligned}$$

Then $\overline{\rho}(\mu)(x+y) \geq \overline{\rho}(\mu)(x) \wedge \overline{\rho}(\mu)(y)$.

(ii)
$$\begin{aligned}\overline{\rho}(\mu)(xy) &= \vee_{r \in [xy]_\rho} \mu(r) \\ &= \vee_{r \in [x]_\rho [y]_\rho} \mu(r) \\ &= \vee_{l \in [x]_\rho,\ m \in [y]_\rho} \mu(lm)\end{aligned}$$

$$\geq \left(\bigvee_{l\in[x]_\rho}\mu(l)\right)\wedge\left(\bigvee_{m\in[y]_\rho}\mu(m)\right)$$
$$=\overline{\rho}(\mu)(x)\wedge\overline{\rho}(\mu)(y).$$
Then $\overline{\rho}(\mu)(xy)\geq\overline{\rho}(\mu)(x)\wedge\overline{\rho}(\mu)(y)$.

Therefore $\overline{\rho}(\mu)$ is a fuzzy subsmiring of R.

Theorem 3.2. *Let ρ be a congruence relation on semiring R and let μ be a fuzzy subset of R. If μ is a fuzzy subsemiring of R, then $\underline{\rho}(\mu)$ is a fuzzy subsemiring of R.*

Proof: Assume that μ be a fuzzy subsemiring of R. Let $x,y\in R$. Then $\mu(x+y)\geq\mu(x)\wedge\mu(y)$ and $\mu(xy)\geq\mu(x)\wedge\mu(y)$. We have

(i)
$$\underline{\rho}(\mu)(x+y)=\bigwedge_{r\in[x+y]_\rho}\mu(r)$$
$$=\bigwedge_{r\in[x]_\rho+[y]_\rho}\mu(r)$$
$$=\bigwedge_{l\in[x]_\rho,\ m\in[y]_\rho}\mu(lm)$$
$$\geq\left(\bigwedge_{l\in[x]_\rho}\mu(l)\right)\wedge\left(\bigwedge_{m\in[y]_\rho}\mu(m)\right)$$
$$=\underline{\rho}(\mu)(x)\wedge\underline{\rho}(\mu)(y).$$
Then $\underline{\rho}(\mu)(x+y)\geq\underline{\rho}(\mu)(x)\wedge\underline{\rho}(\mu)(y)$.

(ii)
$$\underline{\rho}(\mu)(xy)=\bigwedge_{r\in[xy]_\rho}\mu(r)$$
$$=\bigwedge_{r\in[x]_\rho[y]_\rho}\mu(r)$$
$$=\bigwedge_{l\in[x]_\rho,\ m\in[y]_\rho}\mu(lm)$$
$$\geq\left(\bigwedge_{l\in[x]_\rho}\mu(l)\right)\wedge\left(\bigwedge_{m\in[y]_\rho}\mu(m)\right)$$
$$=\underline{\rho}(\mu)(x)\wedge\underline{\rho}(\mu)(y).$$
Then $\underline{\rho}(\mu)(xy)\geq\underline{\rho}(\mu)(x)\wedge\underline{\rho}(\mu)(y)$.

Therefore $\underline{\rho}(\mu)$ is a fuzzy subsemiring of R.

Theorem 3.3. *Let ρ be a congruence relation on R. A fuzzy subset μ of R is a fuzzy bi-ideal of R, then $\overline{\rho}(\mu)$ is a fuzzy bi-ideal of R.*

Proof: Let μ be a fuzzy subsemiring of R. Let $x,y,z\in R$. Then $\mu(x+y)\geq\mu(x)\wedge\mu(y)$, $\mu(xy)\geq\mu(x)\wedge\mu(y)$ and $\mu(xzy)\geq\mu(x)\wedge\mu(y)$. We have

(i)
$$\overline{\rho}(\mu)(x+y)=\bigvee_{a\in[x+y]_\rho}\mu(a)$$
$$=\bigvee_{r\in[x]_\rho+[y]_\rho}\mu(a)$$
$$=\bigvee_{p\in[x]_\rho,\ q\in[y]_\rho}\mu(pq)$$
$$\geq\left(\bigvee_{p\in[x]_\rho}\mu(p)\right)\wedge\left(\bigvee_{q\in[y]_\rho}\mu(q)\right)$$
$$=\overline{\rho}(\mu)(x)\wedge\overline{\rho}(\mu)(y).$$
Then $\overline{\rho}(\mu)(x+y)\geq\overline{\rho}(\mu)(x)\wedge\overline{\rho}(\mu)(y)$.

(ii)
$$\overline{\rho}(\mu)(xy)=\bigvee_{a\in[xy]_\rho}\mu(a)$$
$$=\bigvee_{a\in[x]_\rho[y]_\rho}\mu(a)$$
$$=\bigvee_{p\in[x]_\rho,\ q\in[y]_\rho}\mu(pq)$$
$$\geq\left(\bigvee_{p\in[x]_\rho}\mu(p)\right)\wedge\left(\bigvee_{q\in[y]_\rho}\mu(q)\right)$$
$$=\overline{\rho}(\mu)(x)\wedge\overline{\rho}(\mu)(y).$$
Then $\overline{\rho}(\mu)(xy)\geq\overline{\rho}(\mu)(x)\wedge\overline{\rho}(\mu)(y)$.

(iii)
$$\overline{\rho}(\mu)(xzy)=\bigvee_{a\in[xzy]_\rho}\mu(a)$$
$$=\bigvee_{a\in[x]_\rho[z]_\rho[y]_\rho}\mu(a)$$
$$=\bigvee_{p\in[x]_\rho,r\in[z]_\rho\,q\in[y]_\rho}\mu(prq)$$

$$\geq \left(\bigvee_{p\in[x]_\rho}\mu(p)\right)\wedge\left(\bigvee_{q\in[y]_\rho}\mu(q)\right)$$
$$= \overline{\rho}(\mu)(x)\wedge\overline{\rho}(\mu)(y).$$
Then $\overline{\rho}(\mu)(xzy) \geq \overline{\rho}(\mu)(x)\wedge\overline{\rho}(\mu)(y)$.

Therefore $\overline{\rho}(\mu)$ is a fuzzy bi-ideal of R.

Theorem 3.4. *Let ρ be a congruence relation on semiring R. A fuzzy subset μ of R is a fuzzy bi-ideal of R, then $\underline{\rho}(\mu)$ is a fuzzy bi-ideal of semiring R.*

Proof: Assume that μ be a fuzzy bi-ideal of a semiring R. Let $x,y\in R$. Then $\mu(x+y)\geq\mu(x)\wedge\mu(y)$ and $\mu(xy)\geq\mu(x)\wedge\mu(y)$. We have

(i)
$$\underline{\rho}(\mu)(x+y) = \bigwedge_{a\in[x+y]_\rho}\mu(a)$$
$$= \bigwedge_{a\in[x]_\rho+[y]_\rho}\mu(a)$$
$$= \bigwedge_{p\in[x]_\rho,\ q\in[y]_\rho}\mu(pq)$$
$$\geq \left(\bigwedge_{p\in[x]_\rho}\mu(p)\right)\wedge\left(\bigwedge_{q\in[y]_\rho}\mu(q)\right)$$
$$= \underline{\rho}(\mu)(x)\wedge\underline{\rho}(\mu)(y).$$
Then $\underline{\rho}(\mu)(x+y) \geq \underline{\rho}(\mu)(x)\wedge\underline{\rho}(\mu)(y)$.

(ii)
$$\underline{\rho}(\mu)(xy) = \bigwedge_{a\in[xy]_\rho}\mu(a)$$
$$= \bigwedge_{a\in[x]_\rho[y]_\rho}\mu(a)$$
$$= \bigwedge_{p\in[x]_\rho,\ q\in[y]_\rho}\mu(pq)$$
$$\geq \left(\bigwedge_{p\in[x]_\rho}\mu(p)\right)\wedge\left(\bigwedge_{q\in[y]_\rho}\mu(q)\right)$$
$$= \underline{\rho}(\mu)(x)\wedge\underline{\rho}(\mu)(y).$$
Then $\underline{\rho}(\mu)(xy) \geq \underline{\rho}(\mu)(x)\wedge\underline{\rho}(\mu)(y)$

(iii)
$$\underline{\rho}(\mu)(xzy) = \bigwedge_{a\in[xzy]_\rho}\mu(a)$$
$$= \bigwedge_{a\in[x]_\rho[z]_\rho[y]_\rho}\mu(a)$$
$$= \bigwedge_{p\in[x]_\rho,\, r\in[z]_\rho\, q\in[y]_\rho}\mu(prq)$$
$$\geq \left(\bigwedge_{p\in[x]_\rho}\mu(p)\right)\wedge\left(\bigwedge_{q\in[y]_\rho}\mu(q)\right)$$
$$= \underline{\rho}(\mu)(x)\wedge\underline{\rho}(\mu)(y).$$
Then $\underline{\rho}(\mu)(xzy) \geq \underline{\rho}(\mu)(x)\wedge\underline{\rho}(\mu)(y)$.

Therefore $\underline{\rho}(\mu)$ is a fuzzy bi-ideal of R.

Theorem 3.5. *Let μ be a fuzzy subset of R. Then μ is a fuzzy prime bi-ideal of R if and only if $\mu_t\neq\emptyset$ be a prime bi-ideal of R for every $t\in[0,1]$.*

Proof: Assume that μ is a fuzzy prime bi-ideal of R. Then μ is a fuzzy bi-ideal of R. Assume $\mu_t\neq\emptyset$. By Theorem 2.6, μ_t is a bi-ideal of R. Let $x,y,z\in R$ such that $xzy\in\mu_t$. Since μ is a fuzzy prime bi-ideal of R, $\mu(xzy)=\mu(x)$ or $\mu(xzy)=\mu(y)$. This implies that $x\in\mu_t$ or $y\in\mu_t$. Therefore μ_t is a prime bi-ideal of R.

Conversely assume that for all $t\in[0,1]$, if $\mu_t\neq\emptyset$, then μ_t is a prime bi-ideal of R. Let $x,y,z\in R$. By Theorem 2.6 , μ is a fuzzy bi-ideal of R. This implies that $\mu(xzy)\geq\mu(x)$ and $\mu(xzy)\geq\mu(y)$. Let
$t=\mu(xzy)$. Thus $xzy\in\mu_t$. Since μ_t is a prime bi-ideal of $R, x\in\mu_t$ or $y\in\mu_t$. This implies that $\mu(x)\geq t=\mu(xzy)$ or $\mu(y)\geq t=\mu(xzy)$. Hence $\mu(xzy)=\mu(x)$ or $\mu(xzy)=\mu(y)$. Hence μ is a fuzzy prime bi-ideal of R.

Theorem 3.6. *Let μ be a fuzzy subset of R. Then μ is a fuzzy prime bi-ideal of R if and only if for all $t\in[0,1]$, if $\mu_t^R\neq\emptyset$ then μ_t^R be a prime bi-ideal of R.*

Proof: Assume that μ is a fuzzy prime bi-ideal of R. Then μ is a fuzzy bi-ideal of R. Assume $\mu_t^R \neq \emptyset$. By Theorem 2.6, μ_t^R is a bi-ideal of R. Let $x, y, z \in R$ such that $xzy \in \mu_t^R$. Then $\mu(xzy) > t$. Since μ is a fuzzy prime bi-ideal of R, $\mu(xzy) = \mu(x)$ or $\mu(xzy) = \mu(y)$. This implies that $\mu(x) > t$ or $\mu(x) > t$. Hence $x \in \mu_t^R$ or $y \in \mu_t^R$ Therefore μ_t^R is a prime bi-ideal of R.

Conversely assume that for all $t \in [0,1]$, if $\mu_t^R \neq \emptyset$, then μ_t^R is a prime bi-ideal of R. Let $x, y, z \in R$. By Theorem 2.6 , μ is a fuzzy bi-ideal of R. This implies that $\mu(xzy) \geq \mu(x)$ and $\mu(xzy) \geq \mu(y)$. We have $xzy \in \mu_t^R$ for all $t < \mu(xzy)$. Since μ_t^R is a prime bi-ideal of R for all $t < \mu(xzy), x \in \mu_t^R$ or $y \in \mu_t^R$ for all $t < \mu(xzy)$. This implies that $\mu(x) > t$, or $\mu(y) > t$ for all $t < \mu(xzy)$. Then $\mu(x) \geq \mu(xzy)$ or $\mu(y) \geq \mu(xzy)$. Hence $\mu(xzy) = \mu(x)$ or $\mu(xzy) = \mu(y)$. Hence μ is a fuzzy prime bi-ideal of R.

Let ρ be a congruence relation on R. A fuzzy subset μ of R is called a ρ-lower rough fuzzy prime bi-ideal of R if $\underline{\rho}(\mu)$ is a fuzzy prime bi-ideal of R. A ρ- upper rough fuzzy prime bi-ideal of R is defined analogously. We call $\rho(\mu)$ a rough fuzzy prime bi-ideal of R if it is both ρ-lower and a ρ- upper rough fuzzy prime bi-ideal of R.

Lemma 3.7: *Let ρ be a congruence relation on semiring R. If μ is a fuzzy subset of R. and $t \in [0,1]$, then*

i) $\left(\underline{\rho}(\mu)\right)_t = \underline{\rho}(\mu_t)$,

ii) $(\overline{\rho}(\mu))_t^R = \overline{\rho}(\mu_t^R)$

Proof:

(i)

$$\begin{aligned} x \in \left(\underline{\rho}(\mu)\right)x)_t &\Leftrightarrow \underline{\rho}(\mu)(x) \geq t \\ &\Leftrightarrow \wedge_{y \in [x]_\rho} \mu(y) \geq t \\ &\Leftrightarrow \forall y \in [x]_\rho, \mu(y) \geq t \\ &\Leftrightarrow [x]_\rho \subseteq \mu_t \\ &\Leftrightarrow x \in \underline{\rho}(\mu_t) \end{aligned}$$

Therefore $\left(\underline{\rho}(\mu)\right)_t = \underline{\rho}(\mu_t)$.

(ii)

$$\begin{aligned} x \in (\overline{\rho}(\mu))_t^R(x) &\Leftrightarrow \overline{\rho}(\mu) > t \\ &\Leftrightarrow \vee_{y \in [x]_\rho} \mu(y) > t \\ &\Leftrightarrow \exists y \in [x]_\rho, \mu(y) \geq t \\ &\Leftrightarrow [x]_\rho \cap \mu_t^R \neq \emptyset \\ &\Leftrightarrow x \in \overline{\rho}(\mu_t^R). \end{aligned}$$

Therefore $(\overline{\rho}(\mu))_t^R = \overline{\rho}(\mu_t^R)$.

Theorem 3.8. *Let ρ be a congruence relation on R . A fuzzy subset μ is a fuzzy prime bi-ideal of R, then $\overline{\rho}(\mu)$ is a fuzzy prime bi-ideal of R.*

Proof: Let μ is a fuzzy prime bi-ideal of R. By Theorem 3.6 for all $t \in [0,1]$, if $\mu_t^R \neq \emptyset$ then μ_t is a prime bi-ideal of R. By Theorem [] in [] for all $t \in [0,1]$, if $\mu_t^R \neq \emptyset$, then μ_t^R is a prime bi-ideal of R. From this and Lemma 3.7(ii), for all $t \in [0,1]$, if $\mu_t^R \neq \emptyset$, if $\overline{\rho}(\mu_t^R) \neq \emptyset$, then $\overline{\rho}(\mu_t^R)$ is a prime bi ideal of R. By Theorem 3.6 $\overline{\rho}(\mu)$ is a fuzzy prime bi-ideal of R.

Theorem 3.8. *Let ρ be a congruence relation on R . A fuzzy subset μ is a fuzzy prime bi-ideal of R, then $\underline{\rho}(\mu)$ is a fuzzy prime bi-ideal of R.*

Proof: Let μ is a fuzzy prime bi-ideal of R. By Theorem 3.5 for all $t \in [0,1]$, if $\mu_t \neq \emptyset$ then μ_t is a prime bi-ideal of R. By Theorem [] in [] for all $t \in [0,1]$, if $\mu_t \neq \emptyset$, then μ_t is a prime bi-ideal of

R. From this and Lemma 3.7(i), for all $t \in [0,1]$, if $\mu_t \neq \emptyset$, if $\underline{\rho}(\mu_t) \neq \emptyset$, then $\underline{\rho}(\mu_t)$ is a prime bi ideal of R. By Theorem 3.5 $\underline{\rho}(\mu)$ is a fuzzy prime bi-ideal of R.

Corollary 3.9. . *Let ρ be a congruence relation on R . A fuzzy subset μ is a fuzzy prime bi-ideal of R, then $\rho(\mu)$ is a rough fuzzy prime bi-ideal of R.*

Theorem 3.10. *Let ρ be a congruence relation on R . Then $\overline{\rho}(\mu)$ is a fuzzy prime bi-ideal of R if and only if for ll $t \in [0,1]$, if $\mu_t^R \neq \emptyset$, then $\rho(\mu_t^R)$ prime bi-ideal of R.*

Proof: By Theorem 3.6 and Lemma 3.7(ii) , we can obtain the conclusion easily.

Theorem 3.5. *Let μ be a fuzzy subset of R. Then $\underline{\rho}(\mu)$ is a fuzzy prime bi-ideal of R if and only if for all $t \in [0,1]$, if $\underline{\rho}(\mu_t)$ is a prime bi-ideal of R.*

Proof: By Theorem 3.5 and Lemma 3.7(i), we can obtain the conclusion easily.

3. ROUGH FUZZY PRIME BI-IDEALS IN SEMIRINGS

Definition 4.1. A fuzzy bi-ideal μ of R is called a *rough prime fuzzy bi-ideal* of R if $\underline{\rho}(\mu)$ and $\overline{\rho}(\mu)$ are prime fuzzy bi-ideals of R.

Theorem 4.2. *Let ρ be a congruence relation on R and let μ be a fuzzy subset of R. If μ is a prime fuzzy bi-ideal of R, then $\overline{\rho}(\mu)$ is a prime fuzzy bi-ideal of R.*

Proof. Let μ be a prime fuzzy bi-ideal of R. Then $f \circ g \leq \mu$ implies $f \leq \mu$ or $g \leq \mu$, for any bi-ideals f, g of R. To show that $\overline{\rho}(f) \circ \overline{\rho}(g) \leq \overline{\rho}(\mu)$ implies $\overline{\rho}(f) \leq \overline{\rho}(\mu)$ or $\overline{\rho}(g) \leq \overline{\rho}(\mu)$.

Suppose assume that $\overline{\rho}(f) \geq \overline{\rho}(\mu)$ and $\overline{\rho}(g) \geq \overline{\rho}(\mu)$. Then for $x \in R$

$\overline{\rho}(f)(x) \geq \overline{\rho}(\mu)(x) \Rightarrow \bigvee_{p[x]_\rho} f(p) \geq \bigvee_{p[x]_\rho} \mu(p) \Rightarrow f(p) \geq \mu(p)$ and

$\overline{\rho}(g)(x) \geq \overline{\rho}(\mu)(x) \Rightarrow \bigvee_{p[x]_\rho} g(p) \geq \bigvee_{p[x]_\rho} \mu(p) \Rightarrow g(p) \geq \mu(p)$ which is a contradiction to our assumption.

Thus either $\overline{\rho}(f) \leq \overline{\rho}(\mu)$ or $\overline{\rho}(g) \leq \overline{\rho}(\mu)$.

Therefore $\overline{\rho}(\mu)$ is a prime fuzzy bi-ideal of R.

Theorem 4.3. *Let ρ be a congruence relation on R and let μ be a fuzzy subset of R. If μ is a prime fuzzy bi-ideal of R, then $\underline{\rho}(\mu)$ is a prime fuzzy bi-ideal of R.*

Proof. Similar to Theorem 4.2.

Proposition.4.4.

i) *Every strongly prime fuzzy bi-ideal of R is a rough prime fuzzy bi-ideal of R.*
ii) *Every prime fuzzy bi-ideal of R is a rough semiprime fuzzy bi-ideal of R.*
iii) *he intersection of any family of prime fuzzy bi-ideals of R is a rough semiprime fuzzy ideals of R.*

Proof: Straightforward.

4.CONCLUSION

The theory of semirings has wide applications in several areas such as optimization theory, discrete event dynamical systems, automata theory, formal language theory and parallel computing. The theory of fuzzy sets and rough sets also has many applications in the above areas. In this paper, we developed the concept of a rough fuzzy prime bi-ideal rough prime fuzzy bi-ideals, rough semiprime fuzzy bi-ideals rough irreducible fuzzy bi-ideals, rough strongly irreducible fuzzy bi-ideals of a semiring. We certainly hope that our work will be very useful both in the theoretical and application aspect. We also propose to work further on this area to bring out many more interesting properties of rough fuzzy prime bi-ideals in rings.

REFERENCES

1) S. Bashir, J. Mehmood, M. Shabir, *Prime Bi-Ideals And Prime Fuzzy Bi-Ideals In Semirings*, World applied Sciences Journal, 22, (2013), pp. 106-121.
2) R. Biswas and S. Nanda, *Rough Groups And Rough Subgroups*, Bulietin polish Academy Science Mathematics 42(1994) 251-254.
3) R. Chinram, *Rough Prime Ideals And Rough Fuzzy Prime Ideals In Γ- Semigroups* Communication of the Koren Mathematical Society,24(3)(2009)341-351.
4) R. Chinram, *A Note On (m,n)- Quasi-Ideals In Semirings*, International Journal of Pure and Applied Mathematics, 49(1), (2008), pp. 45-52.
5) R. Chinram and C. Jirojkul, *On Bi-Ideals In Γ -Semigroups*, Journal of science and Technology 29(1)(2007) 231-234.
6) B. Davvaz, *Roughness In Rings*, Information Sciences, 164, (2004), pp. 147-163.
7) D. Dubois and H. Parade, *Rough Fuzzy Sets And Fuzzy Rough Sets*, International Journal of General System, 17(2-3), (1990), 191-209.
8) T.K. Dutta and B.K. Biswas, *Fuzzy Prime Ideals Of A Semiring*, Bulletin Malaysian Mathematical Society, 17,(1994), 9-16.
9) O.Kanhi and B. Davvaz, *On The Structure Of Rough Prime (Primary) Ideals And Rough Fuzzy Prime (Primary) Ideals In Commutative Rings*, Information Sciences, 178(5), (2008), pp. 1343-1354.
10) W.J.Liu, *Fuzzy Invarient Subgroups And Fuzzy Ideals*, Fuzzy Sets And Systems, 8(2)(1982), 133-139.
11) Z. Pawlak, *Rough Sets*, International Journal of Information Computer Science, 11 (1982) 341-356.
12) Z. Pawlak, *Rough Sets And Fuzzy Sets*, Fuzzy sets and systems, 17(1)(1985)99-102.
13) Z. Pawlak and A.Showron, *Rough Sets:Some Extensions*, Information Sciences, 177(1) (2007) 28-40.
14) Z. Pawlak, *Rough sets-Theoretical aspects of reasoning about data*, Kuluwer Academic publishers, Dordrecht(1991).
15) A. Rosenfeld, *Fuzzy Groups*, Journal of Mathematics Annals Applications, 35(1971), 512-517.
16) G.Senthil Kumar, V.Selvan, *The Lower and Upper Approximation of Fuzzy Ideals in Semirings*, International Journal of Science and Engineering Research, Volume 3, Issue 1, January 2012, 1-5.
17) V.S.Subha, *Rough k-ideals in Semirings*, International Journal of Research Publication & Seminar, 5, Issue 1 March, 2014, 117-126
18) V.S.Subha, *Rough Quasi-ideals in Regular Semirings*, International Journal of Research Publication & Seminar, Volume 06 (01), February, 2015, 116-123.
19) V.S. Subha, *Rough Prime ideals in in Γ-semirings*, Universal Research Reports , Volume 4, Issue 6, July- September 2017, 149-155.
20) N. Thillaigovindan, V. Chinnadurai and V.S. Subha, *Rough ideals in Γ- semigroups*, International Journal of Algebra, Vol. 6, 2012, no. 14, 651-661.
21) H.S. Vandiver, *Note On A Simple Type Of Algebra In Which The Cancellation Law Of Addition Does Not Hold*, Bulletin American Mathematical Society, (1934), pp. 916-920.
22) L.A.Zadeh, *Fuzzy Sets*, Information Control, 8, (1965) 338-353.