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A STUDY OF DIFFERENCES BETWEEN SEQUENCE AND SERIES WITH VARIOUS REAL APPLICATIONS

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ABSTRACT
In this study, we examine the differences between real analysis concepts- sequence and series. This is accomplished by conducting textbook analysis commonly used in real analysis textbooks. The differences discussed through different real life examples and applications. Arithmetic and geometric progression are studied in depth with the help of examples. Here we learn interesting methods of finding the nth term and partial sum for series. The results of this textbook analysis suggests that several differences between sequence and series do exist with a particular emphasis on functions. Implications, limitations, and future research are also discussed.

Keywords:
Real Analysis, definition of sequence and series, Real life examples, Arithmetic and Geometric Applications of sequence and series, Real life importance.

INTRODUCTION

Real analysis is a branch of mathematical analysis that analyses the behaviour of real numbers, sequences and series, and real functions. Convergence, limits, continuity, smoothness, differentiability, and inerrability are some of the features of real-valued sequences and functions that real analysis explores. In this research article, let us discuss the brief introduction about the differences between sequence and series.

In mathematics, informally speaking, a sequence is an ordered list of objects (or events). Like a set, it contains members (also called elements, or terms). The number of ordered elements (possibly infinite) is called the length of the sequence. Unlike a set, order matters, and exactly the same elements can appear multiple times at different positions in the sequence. Most precisely, a sequence can be defined as a function whose domain is a countable totally ordered set, such as the natural numbers.

For example, (M, A, R, Y) is a sequence of letters with the letter 'M' first and 'Y' last. This sequence differs from (A, R, M, Y). Also, the sequence (1, 1, 2, 3, 5, 8), which contains the number 1 at two different positions, is a valid sequence. Sequences can be finite, as in this example, or infinite, such as the sequence of all even positive integers (2, 4, 6,...). Finite sequences are sometimes known as strings or words and infinite sequences as streams. The empty sequence ( ) is included in most notions of sequence, but may be excluded depending on the context.

Examples and notation

A sequence can be thought of as a list of elements with a particular order. Sequences are useful in a number of mathematical disciplines ... In particular; sequences are the basis for series, which are important in differential equations and analysis. Sequences are also of interest in their own right and can be studied as patterns or puzzles, such as in the study of prime numbers.

There are many important integer sequences. The prime numbers are numbers that have no divisors but 1 and themselves. Taking these in their natural order gives the sequence (2,3,5,7,11,13,17,...). The study of prime numbers has important applications for mathematics and specifically number theory.

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The Fibonacci numbers are the integer sequence whose elements are the sum of the previous two elements. The first two elements are either 0 and 1 or 1 and 1 so that the sequence is (0,1,1,2,3,5,8,13,21,34,...).
Sequence and Series is one of the important topics in Mathematics. Though many students tend to get confused between the two, these two can be easily differentiated. Sequence and series can be differentiated, in which the order of sequence always matters in the sequence but it's not the case with series.
Sequence and series are the two important topics that deal with the listing of elements. It is used in the recognition of patterns, for example, identifying the pattern of prime numbers, solving puzzles, and so on. Also, the series plays an important role in the differential equations and in the analysis process. In this article, let us discuss the key difference between the sequence and series in detail. Before that, we will see a brief definition of the sequence and series.

OBJECTIVES OF THE STUDY

1. To get an understanding about sequence and series
2. To get an understanding about real life importance of sequence and series
3. To get an understanding about applications of sequence and series

DEFINITION OF SEQUENCE AND SERIES IN MATHEMATICS

Sequence

The sequence is defined as the list of numbers that are arranged in a specific pattern. Each number in the sequence is considered a term. For example, 5, 10, 15, 20, 25,
... is a sequence. The three dots at the end of the sequence represents that the pattern will continue further. Here, 5 is the first term, 10 is the second term,15 is the third term and so on. Each term in the sequence can have a common difference, and the pattern will continue with the common difference. In the example given above, the common difference is 5. The sequence can be classified into different types, such as:
Arithmetic Sequence ,Geometric Sequence, Harmonic Sequence, Fibonacci Sequence.

Series

The series is defined as the sum of the sequence where the order of elements does not matter. It means that the series is defined as the list of numbers with the addition symbol in between. The series can be classified as a finite series or infinite series which depends on the type of sequence whether it is finite or infinite. Note that, the finite series is a series where the list of numbers has an ending, whereas the infinite series is never-ending. For example, 1+3+5+7+.. is a series.
The different types of series are:

1. Geometric series
2. Harmonic series
3. Power series
4. Alternating series
5. Exponent series (P-series)

REAL-LIFE EXAMPLES FOR A SEQUENCE AND A SERIES

Suppose I'm eating a packet of cookies. At the first time I eat 1 cookie, and 1⁄2 at the second time. And every time afterwards, I'll eat half amount of the previous.
The amount of cookies I eat every time is a sequence made by a list of numbers. This sequence converges to 0, because I eat less and less cookies.

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The total amount of cookies I eat is a series, which sums up how much I ate. This series converges because it is a geometric series with a ratio of 1⁄2. And series ∑(0.5)^n converges to 2.

Sequence

Sequence and Series is one of the most important concepts in Arithmetic. A sequence refers to the collection of elements that can be repeated in any sort. In other words, we can say a sequence refers to the list of objects or items that have been arranged in a systematic and sequential manner.

Eg: a1,a2,a3, a4…….

Series

Whereas, Series refers to the sum of all the elements available or we can say A series can be referred to as the sum of all the elements available in the sequence. One of the most common examples of a sequence and series would be Arithmetic Progression.

Eg: If a1,a2,a3, a4……. etc is considered to be a sequence, then the sum of terms in the sequence a1+a2+a3+ a4……. are considered to be a series.

TYPES OF SEQUENCES

The main types of sequences are discussed here:

1. **ARITHMETIC SEQUENCE**

Consider the following sequence for instance, 1,4,7,10,13,16…….

Here, we can see that each term is obtained by adding 3 to the preceding term. This sequence is called Arithmetic sequence or Arithmetic Progression. It is also abbreviated as

A.P. Thus we can define Arithmetic Sequence as

A sequence x1,,x2,x3,......xn. Is called an Arithmetic Progression, if there exists a constant number m such that

x2 = x1+ m x3 = x2+ m x4= x3+ m

.

.

xn = xn-1+ m and so on

The constant m is called the common difference of the A.P. Thus we can write it as, Where x is the first term, m is a common difference.

2. **GEOMETRIC SEQUENCE**

A sequence in which each term is obtained by either multiplying or dividing a certain constant number with the preceding one is said to be a geometric sequence.

For example: 2,4,8,16,32,64,128...and so on

Here we can see that there is a common factor 2 between each term.

The geometric sequence can be commonly written as, Where a is the first term, m is the common factor between the terms.

A few other types of Sequence and Series include:

3. **HARMONIC SEQUENCES**

Harmonic Sequences refer to a series of numbers that are said to be in a harmonic sequence. These series of numbers are said to be in a harmonic sequence only if the reciprocal of all the elements that are a part of the sequence are created into an arithmetic sequence.

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4. FIBONACCI NUMBER SEQUENCE

Fibonacci Numbers are a form of a number sequence where every element can be obtained by adding two elements. With this, the sequence starts with 0 and 1. Hence, the Fibonacci Number Sequence is defined as
F1=0, F2 = 1,
F3 = 1,
F4 = 2,
F5 = 3,
…
..Fn = Fn-1 + Fn-2

Differences Between Sequence and Series

There is a bit of confusion between sequence and series, but we can easily differentiate between Sequence and series as follows:

1. A sequence is a particular format of elements in some definite order, whereas a series is the sum of the elements of the sequence. In sequence order of the elements are definite, but in series the order of elements is not fixed.
2. A sequence is represented as1,2,3,4,....n, whereas the series is represented as 1+2+3+4+ n.
3. In sequence, the order of elements has to be maintained, whereas in series the order of elements is not important.

SEQUENCE AND SERIES FORMULAS

Formulas for Arithmetic Sequence:

Sequence = x, x+m, x+2m, x,+3m, x+(n-1)m
Where x is the first term m is a common difference.
Common difference = m = Successive term - Preceding term = x2- x1 General term = nth term = x+(n-1)m
Sum of first nth terms =sn = n2(2a+(n−1)d)

Formulas for Geometric Sequence:

Sequence = a, am, am2, am3…….
Where a is the first term, m is the common factor between the terms. Common factor = m = Successive term / Preceding term m = am(n−1)am(n−2) General term = nth term = an = am(n−1)
Sum of n terms in GP = Sn = na if m = 1,
Sum of n terms in GP = Sn = a(mn−1)(m−1) when m > 1 Sum of n terms in GP = Sn = a(1−mn)(1−m) when m < 1

Formula for Series:

sn=n2(1+n)

APPLICATIONS OF SEQUENCES & SERIES

1. Applications of Arithmetic Sequences & Series

Many real-life situations can be modelled using sequences and series, including but not limited to: patterns made when tiling floors; seating people around a table; the rate of change of a population; the spread of a virus and many more.
What do we need to know about applications of arithmetic sequences and series?
If a quantity is changing repeatedly by having a fixed amount added to or subtracted from it then the use of arithmetic sequences and arithmetic series is appropriate to model the situation

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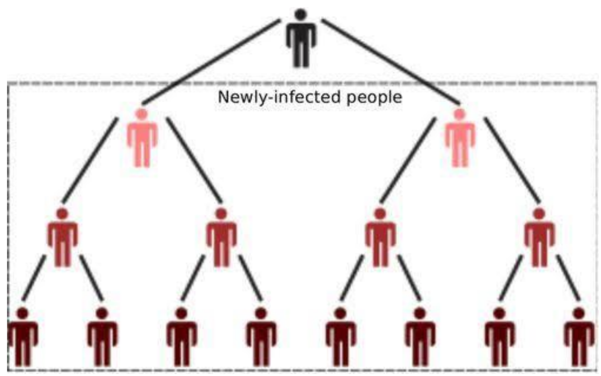
If a sequence seems to fit the pattern of an arithmetic sequence it can be said to be modelled by an arithmetic sequence
A common application of arithmetic sequences and series is simple interest
Simple interest is when an initial investment is made and then a percentage of the initial investment is added to this amount on a regular basis (usually per year)
The scenario can be modelled using the given information.
Arithmetic sequences can be used to make estimations about how something will change in the future.

2. **Applications of Geometric Sequences & Series**



In simple words, if a quantity is changing repeatedly by a fixed percentage, or by being multiplied repeatedly by a fixed amount, then the use of geometric sequences and geometric series is appropriate to model the situation.
If a sequence seems to fit the pattern of a geometric sequence it can be said to be modelled by a geometric sequence
The scenario can be modelled using the given information and the formulae from the formulas. A common application of geometric sequences and series is compound interest. Compound interest is when an initial investment is made and then interest is paid on the initial amount and on the interest already earned on a regular basis (usually every year) Geometric sequences can be used to make estimations about how something will change in the future

REAL LIFE IMPORTANCE OF SEQUENCE AND SERIES

It's always better to know how knowledge helps us in real life. If you look around in your surroundings, you will find a number of patterns in nature – leaves and flowers with similar structures, the ripples on a lake, the symmetry of a starfish and many more patterns that don't cease to amaze us. Nature inspired mathematicians to try and explain these patterns in nature; to work on mathematical models and understand the basics of geometric shapes and structures. Lot of work has been carried out in the field of number sequences and series to predict the possibility of an event, designing structures and buildings, analysis of real-life situations etc.
Mathematical sequences and series are also used in business and financial analysis to assist in decision-making and find the best solution to a given problem. Organizations use quantitative analysis in risk assessment and management, making investment decisions, pricing and many more important functions. If you are a business analyst, a statistician or an investment manager, your work will revolve around number

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patterns and the analysis of these patterns. So, let's try and understand what exactly these numbers are and why are they so special?

What are Sequences and Series?

A Sequence is simply defined as a set of numbers that are in a particular order. For example, a sequence of even numbers will be (2, 4, 6, 8 ...). If a sequence goes on forever, like the one mentioned, then it's called an infinite sequence. However, if we say, 'sequence of the first four even numbers', then it will be a finite sequence that looks like this – (2, 4, 6, and 8)

A Series, on the other hand is the sum total of the numbers in a sequence and they too will be either infinite or finite in nature. In our above given example, the finite series will be the Summation ∑ (2+4+6+8) whereas the infinite series will be the Summation ∑ (2+4+6+8+...).

Examples of Sequences and Series

There are numerous mathematical sequences and series that arise out of various formulas. The arithmetical and geometric sequences that follow a certain rule, triangular number sequences built on a pattern, the famous Fibonacci sequence based on recursive formula, sequences of square or cube numbers etc. Series like the harmonic series, alternating series, Fourier series etc. have great importance in the field of calculus, physics, analytical functions and many more mathematical tools. They are widely used to in computer science, engineering, finance and economics etc. to determine various possibilities of a certain situation or criteria to design, analyze, build or predict something.

Importance of Sequences and Series

Did you know that the behavior of your company stocks in the financial markets follow a certain sequence? Or for that matter, you can predict how long it will take for your financial assets to double in market value? With the help of number sequences and series formulas, you can perform Quantitative Analysis, Financial and Business Analysis to help you in important business and investment decisions

CONCLUSION

A sequence is a particular format of elements in some definite order, whereas a series is the sum of the elements of the sequence. In sequence order of the elements are definite, but in series the order of elements is not fixed.

- In a maths series and sequence, a is considered to be the first term, d is the common difference, and an is known to be the nth term or the last term.
- The arithmetic sequence can be explained as a, a + d, a + 2d, a + 3d, …
- Each term in a geometric progression is obtained by multiplying the common ratio of the successive term to the preceding term.
- A series is the sum of the terms in a sequence. The sum of the first n terms is called the nth partial sum and is denoted by Sn

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