Study of Fixed Point Theorems in Fuzzy Metric Spaces

Rekha Rani, Research Scholar, Deptt. Of Mathematics
NIILM University, Kaithal

Abstract : Fixed point theorems are the most important tools for providing existence and uniqueness of the solution of various mathematics models (different, integral, partial differential equation and variational inequalities) representing different phenomena suitable for different fields such as steady state temperature distribution, chemical reaction, neutron transport theory, economic theories, epidemics and flow of fluids. At present this field has been recognized as one of the active field of research.

Fixed point theorems is one of the most fruitful and effective tools in mathematics which has enormous applications within as well as outside the mathematics. Despite noted improvements in computer skill and its remarkable success in facilitating many areas of research, there will stands one major short coming: computers are not designed to handle situations wherein uncertainties are involved. To deal with uncertainty. We need techniques other than classical ones wherein some specific logic is required.

Introduction : In 1922, S. Banach put forward the first fixed point theorem in metric space for contraction mapping. Contraction mapping gives rise to several other mappings, namely contractive, no expansive, LipchitZ's etc. and all of them are continuous. Nearly four decades after introduction of Banach's fixed point theorem, M.Edelstein (1961) made an extensive generalization of it and established a class of new fixed point theorems for a special class of mappings in metric spaces.

Since then, a number of generalization of contraction mapping principle have been established by different mathematicians leading to a volume of fixed point theorems in metric spaces and it is continued till now. Of course, there are other fixed point theorems such as J.Caristi's (1975, 1976) fixed point theorem related to arbitrary mapping.

With the introduction of the concept "Fuzzy Set" by Lofty A. Zadeh in 1965, new horizon was opened in the sky of human knowledge. His way of defining 'Fuzziness' in closely related to process of human thinking and reasoning and so is a tool for generating decisions in uncertainty. In one word, fuzzy set provides scope to express uncertainty or vagueness suitably in a mathematical way. Gradually fuzzy set theory has entered into almost all the disciplines of science, technology and humanities and now a days it is an extremely versatile interdisciplinary

research area. People have been experiencing the fruits of application of fuzzy logic and fuzzy set theory at large by adopting them everywhere from household appliances to traffic controlling system of high speed train. Fuzzy set theory is also widely used in information technology and resource management analysis. Every application of fuzzy logic can realize some benefits such as performance, simplicity, lower cost, productivity.

The fuzzy concept in metric space was introduced for the first time in 1975 by O.Kramosil and J.Michalek generalizing the concept of probabilistic metric space in fuzzy situation. A George and P.Veerarnani (1994) modified the notion of fuzzy metric space with the help of continuous t-norm and defined the Hausdorff topology of fuzzy metric space which have very important application in quantum particle physics particularly in connections with both string and theory which were given and studied by E.I. Naschie. in 1979, M.A Ecreg introduced a definition of fuzzy metric space using the concept of lattices. In 1981, S. Heippern developed the notion of fuzzy mapping and proved a fixed point theorem for fuzzy mapping. Z. Deng, in 1982 introduced fuzzy pseudo metric space. In 1984, 0. Kaleva and S. Seikkala introduced the notion of fuzzy . metric by setting the distance between two point to be a non negative fuzzy number. Ordering and triangular inequality of fuzzy number were also defined. The duo gave a new turn to a — level set introduced by L.A. Zadeh. On the basis of a —level set they discussed some properties of fuzzy numbers and fuzzy metric space and accordingly explained the existence of a Hausdoiff topology in fuzzy metric space. Under certain restrictions on the fuzzy metric restrictions on the fuzzy metric space, this Hausdorff topology gives a Hausdroff uniformity for which a fuzzy metric space can be represented as fuzzy uniform space. Many researchers have obtained common fixed point theorems for mapping satisfying different types of commutativity conditions. B. Singh and M.S. Chauhan and R. Vasuki introduced the concept of R — weakly commuting and compatible mappings, respectively in fuzzy metric space. S. Sessa presented a generalization of the concept of commutativity, called weak commutativity of mappings in fixed — point consideration in metric spaces. A much more general concept than that of weak commutativity, namely "weak compatibility" developed by G.Jungck and B.E.Rhoades and proved some fixed point theorems for such mappings without appeal to continuity in metric space.

We keep it mind that if the distance between objects is fuzzy, then the object may or may not be fuzzy. That is in fuzzy metric space the set will be fuzzy, but in fuzzy 2-metric space the distance between objects with respect to the nearness function will be fuzzy, while the set may or

may not be fuzzy. The interesting results in this direction are come from a series of papds by S.Gahler, who investigated 2-metric space. P.L.Sharma and K.Iseki studies the first time contraction type mapping in 2-metric space. Recently Z.Wenzhi and others initiated the study of 2-PM spaces. We know that 2-metric space is a real valued function of a part triples on a set X, which abstract properties were suggested by the area function in Euclidian spaces. Now it is natural to expect 3-metric space, which is suggested by the volume function.

Conclusion :

The aim of the present paper is to study the fixed point theorems in complete and compact fuzzy metric spaces as improvement of some recent results For this purpose, the condition of the maximum type defined by altering distance is used. The research is illustrated by these complex.

The concept of fuzzy metric spaces was introduced initially by Kramosil and Michalek. Later on, George and Veeramai modified the notion of fuzzy metric spaces due to kramosil and Michalek and studied a Hausdorff topology of fuzzy metric spaces.

Main aim of this paper is to prove some fixed point theorems in Fuzzy Metric spaces through rational inequality. Our results extends and generalizes the results of many other authors existing in the literature. Some applications are also given in support of our results.

Main aim of this gaper is to prove some fixed point theorems in Fuzzy Metric spaces through rational inequality. Our results extends and generalizes the results of many other authors existing in the literature. Some applications are also given in support of our results.

The foundation of fuzzy mathematic is laid by Lifti..A Zadeh with the introduction of fuzzy sets in 1965. This foundation represents a vagueness in everyday life. Subsequently several authors have applied various form general topology of fuzzy sets and developed the concept of fuzzy space. In 1975, Kramosil and Michalek introduced concept of fuzzy metric spaces.

References:

1. M.Abbas, M. Imdad, D. Gopal, Phi-weak contractions in fuzzy metric spaces, Iran. J. Fuzzy Syst. D (2010).

2. Dan Butnariu, A fixed point theorems for point to fuzzy set mappings, Notices Am. Math. Soc., (1979).