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# Study of The Principle of Minimum Potential Energy

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**Abstract :** Deformation and stress analysis of structural systems can be accomplished using the principle of Minimum Potential Energy (MPE), which states that For conservative structural systems, of all the kinematically admissible deformations, those corresponding to the equilibrium state extremize (i.e., minimize or maximize) the total potential energy. If the extremum is a minimum, the equilibrium state is stable.

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Let us first understand what each term in the above statement means and then explain how this principle is useful to us. A constrained structural system, i.e., a structure that is fixed at some portions, will deform when forces are applied on it. Deformation of a structural system refers to the incremental change to the new deformed state from the original undeformed state. The deformation is the principal unknown in structural analysis as the strains depend upon the deformation, and the stresses are in turn dependent on the strains.

Therefore, our sole objective is to determine the deformation. The deformed state a structure attains upon the application of forces is the equilibrium state of a structural system. The Potential energy (PE) of a structural system is defined as the sum of the strain energy (SE) and the work potential (WP).

$$PE = SE + WP \tag{1}$$

The strain energy is the elastic energy stored in deformed structure. It is computed by integrating the strain energy density (i.e., strain energy per unit volume) over the entire volume of the structure.

$$SE = \int_V (strain\,energy\,density)\,dV \tag{2}$$

The strain energy density is given by

$$Strain\ energy\ density = \frac{1}{2}(stress)(strain) \tag{2a}$$

The work potential WP, is the negative of the work done by the external forces acting on the structure. Work done by the external forces is simply the forces multiplied by the displacements at the points of application of forces. Thus, given a deformation of a structure, if we can write down the strains and stresses, we can obtain SE, WP, and finally PE. For a structure, many deformations are possible. For instance, consider the pinned-pinned beam shown in Figure 1a. It can attain many deformed states as shown in Figure 1b. But, for a given force it will only attain a unique deformation to achieve equilibrium as shown in Figure 1c. What the principle of MPE implies is that this unique deformation corresponds to the extremum value of the MPE. In other words, in order to determine the equilibrium deformation, we have to extremize the PE. The extremum can be either a minimum or a maximum. When it is a minimum,

the equilibrium state is said to be stable. The other two cases are shown in Figure 2 with the help of the classic example of a rolling ball on a surface.

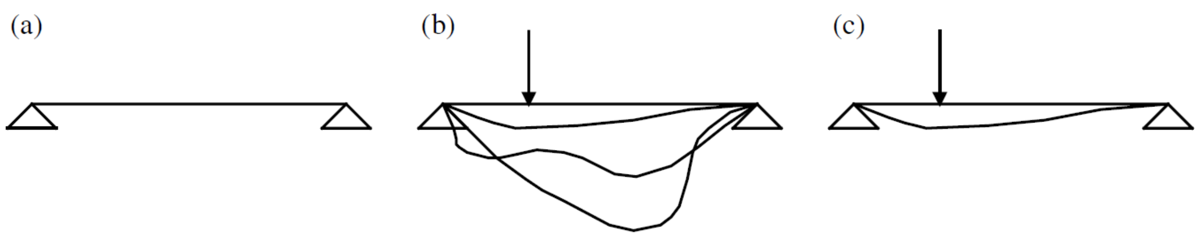


**Figure 1 The notion of equilibrium deformed state of a pinned-pinned beam**

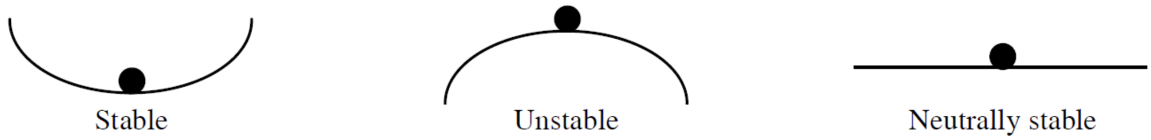


**Figure 2 Three equilibrium states of a rolling ball**

There are two more new terms in the statement of the principle of MPE that we have not touched upon. They are conservative system and the kinematically admissible deformations. Conservative systems are those in which WP is independent of the path taken from the original state to the deformed state. Kinematically admissible deformations are those deformations that satisfy the geometric (kinematic)
boundary conditions on the structure. In the beam example above , the boundary conditions include zero displacement at either end of the beam.

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