



A Survey of Transportation Problem For Numerical Solution of Mathematical problems

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ABSTRACT : *The optimization processes in mathematics, computer science and economics are solving effectively by choosing the best element from set of available alternatives elements. The most important and successful applications in the optimaization refers to transportation problem (TP), that is a special class of the linear programming (LP) in the operation research (OR). The main objective of transportation problem solution methods is to minimize the cost or the time of transportation. Most of the currently used methods for solving transportation problems are trying to reach the optimal solution, whereby, most of these methods considerd complex and very expansive in term of the execution time. In this study use the best candidate method (BCM), in which the key idea is to minimize the combinations of the solution by choosing the best candidates to reach the optimal solution. There is a hello, how r u? Problems involving transporting products from several sources to several destinations. Although the formation can be used to represent more general assignment and scheduling problems as well as transportation and distribution problems. The two common objectives of such problems are either (1) minimize the cost of shipping m units to n destinations or (2) maximize the profit of shipping m units to n destinations. Comparatively, applying the BCM in the proposed method obtains the best initial feasible solution to a transportation problem and performs faster than the existing methods with a minimal computation time and less complexity. The proposed methods are therefore an attractive alternative to traditional problem solution methods. The BCM can be used successfully to solve different business problems of distribution products that are commonly referred to transportation problems.*

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KEYWORDS: : *Operation research, transportation problem, linear programming, optimization problems, transportation model, Vogel's approximation method, best candidate's method, optimal solution, solving optimization, and objective function.*

INTRODUCTION: Down through The problem of minimizing the total cost of transportation has been studied since long and is well known. In a time minimizing transportation problem, the time of transporting goods is minimized to satisfy certain conditions in respect of availabilities at sources and requirements at destinations. The basic difference between cost minimizing and time minimizing transportation problem is that the cost of transportation changes with variations in the quantity but the time involved remains unchanged and irrespective of the quantities. The time minimizing transportation problem has been studied by Hammer [1, 2], Garfinkel and Rao [3], Szwarc [4], Bhatia, Swarup and Puri [5], Ramakrishnan [6], Sharma and Swarup [7], Seshan and Tikekar [8] and by several other authors. Time-cost trade off means the problem of minimizing the transportation cost in addition to minimizing the time of the transportation. Time-cost trade off analysis has been discussed by Satya Prakash [9], Bhatia, Swarup and Puri [10] and several other authors. Satya Prakash [9] has used goal programming approach to solve the problem. Liu [11] discussed a method for solving the cost minimization transportation problem with varying demand and supply. Most of the models developed for solving the transportation problem are with the assumption that the supply, demand and the cost per unit values are exactly known. But in real world applications, the supply, the demand and the cost per unit of the quantities are generally not specified precisely i.e. the parameters are fuzzy in nature. Impreciseness in the parameters means the information for these parameters are not complete. But even with incomplete information, the model user is normally able to give a realistic interval for the parameters. Carlsson and Korhonen [12] and Chanas [13] discussed parametric approach to deal with the fuzzy parameters. In this paper cost and time minimization transportation problem has been dealt when the supply, the demand and the transportation cost per unit of the quantities are fuzzy. Impreciseness in the parameters has been dealt with using the concept developed in Ref. [12] and for the time minimization; the set of times (required for different sources to different destinations) is partitioned suitably. Linear and exponential membership functions have been considered for the imprecise parameters. The problem has been posed as multi objective linear programming problem and solved using preemptive goal programming approach by assigning different priorities to the different objectives. The model intends to obtain a compromise solution with minimum transportation cost as well as minimum transportation time.



Transportation problem is concerned with the optimal pattern of the product units' distribution from several original points to several destinations. Suppose there are m points of original $A_1, \dots, A_i, \dots, A_m$ and n destinations $B_1, \dots, B_j, \dots, B_n$. The point $A_i (i = 1, \dots, m)$ can supply a_i units, and the destination $B_j (j = 1, \dots, n)$ requires b_j units.

Let us assume there are m sources supplying n destinations. Source capacities, destinations requirements and costs of material shipping from each source to each destination are given constantly. The transportation problem can be described using following linear programming mathematical model and usually it appears in a transportation tableau. There are three general steps in solving transportation problems. We will now discuss each one in the context of a simple example. Suppose one company has four factories supplying four warehouses and its management wants to determine the minimum-cost shipping schedule for its weekly output of chests. Factory supply, warehouse demands, and shipping costs per one chest (unit) are shown in Table 1

				Shipping Cost per Unit (in \$)				
Factory	Supply	Warehouse	Demand	From	To E	To F	To G	To H
A	15	E	10	A	10	30	25	15
B	6	F	12	B	20	15	20	10
C	14	G	15	C	10	30	20	20
D	11	H	9	D	30	40	35	45

Table 1 "Data for Transportation Problem"

At first, it is necessary to prepare an initial feasible solution, which may be done in several different ways; the only requirement is that the destination needs be met within the constraints of source supply.

MATERIALS AND METHOD: Transportation model: In a transportation problem, we are focusing on the original points. These points may represent factories to produce items, and to supply a required quantity of the products to a certain number of destinations. This process must be done successfully in such a way as to maximize the profit or minimize the cost transportation. Therefore, the places of production and supply are collected as the original points and the destinations respectively. Sometimes the original and destinations points are also termed as sources and sinks. However, to illustrate a typical transportation model, suppose that m factories supply certain items to n warehouses. As well as, let factory $i (i = 1, 2, m)$ produces a_i units, and the warehouse $j (j = 1, 2, \dots, n)$ requires b_j units. Furthermore, suppose the cost of transportation from factory i to warehouse j is c_{ij} . The decision variables x_{ij} is being the transported amount from the factory i to the warehouse j . typically, our objective is to find the transportation pattern that will minimize the total of the transportation cost.

The Transportation Matrix: The transportation matrix for this example appears in Table 2, where supply availability at each factory is shown in the far right column and the warehouse demands are shown in the bottom row. The unit shipping costs are shown in the small boxes within the cells (see transportation tableau – at the initiation of solving all cells are empty). It is important at this step to make sure that the total supply availabilities and total demand requirements are equal. Often there is an excess supply or demand. In such situations, for the transportation method to work, a dummy warehouse or factory must be added. Procedurally, this involves inserting an extra row (for an additional factory) or an extra column (for an ad warehouse). The amount of supply or demand required by the "dummy" equals the difference between the row and column totals. In this case there is: Total factory supply ... 51, Total warehouse requirements ... 52. This involves inserting an extra row - an additional factory. The amount of supply by the dummy equals the difference between the row and column totals. In this case there are $52 - 51 = 1$. The cost figures in each cell of the dummy row would be set at zero so any units sent there would not incur a transportation cost. Theoretically, this adjustment is equivalent to the simplex procedure of inserting a slack variable in a constraint inequality to convert it to an equation, and, as in the simplex, the cost of the dummy would be zero in the objective function.



From	To	E	F	G	H	Factory Supply
A		10	30	25	15	14
B		20	15	20	10	10
C		10	30	20	20	15
D		30	40	35	45	12
Dummy		0	0	0	0	1
Destination Requirements		10	15	12	15	52

Table 2 "Transportation Matrix for Chests Problem With an Additional Factory (Dummy)"

Algorithms for solving: there are several algorithms for solving transportation problems which are based on different of special linear programming methods, among these are:

1. Northwest Corner method
2. Minimum cost method
1. Genetic algorithm
3. Vogel's approximation method
4. Row Minimum Method
5. Column Minimum Method

Basically, these methods are different in term of the quality for the produced basic starting solution and the best starting solution that yields smaller objective value. In this study, we used the Vogel's approximation method, since it generally produces better starting solutions than other solving methods; as well we have used the BCM solution steps [8].

The Best Candidates Method (BCM): BCM process includes three steps; these steps are shown as follows [8]:

Step1: Prepare the BCM matrix, If the matrix unbalanced, then the matrix will be balanced without using the added row or column candidates in solution procedure.

Step2: Select the best candidates that are for minimizing problems to the minimum cost, and maximizing profit to the maximum cost. Therefore, this step can be done by electing the best two candidates in each row. If the candidate repeated more than two times, then the candidate should be elected again. As well as , the columns must be checked such that if it is not have candidates so that the candidates will be elected for them. However, if the candidate is repeated more than one time, the elect it again.

Step3: Find the combinations by determining one candidate for each row and column, this should be done by starting from the row that have the least candidates, and then delete that row and column. If there is situation that has no candidate for some rows or columns, then directly elect the best available candidate. Repeat Step 3 by determining the next candidate in the row that started from. Compute and compare the summation of candidates for each combination. This is to determine the best combination that gives the optimal solution.

Initial Feasible Solution: Initial allocation entails assigning numbers to cells to satisfy supply and demand constraints. Next we will discuss several methods for doing this: the Northwest-Corner method, Least-Cost method, and Vogel's approximation method (VAM). Table 3 shows a northwest-corner assignment. (Cell A-E was assigned first, A-F second, B-F third, and so forth.) Total cost: $10 \cdot 10 + 30 \cdot 4 + 15 \cdot 10 + 30 \cdot 1 + 20 \cdot 12 + 20 \cdot 2 + 45 \cdot 12 + 0 \cdot 1 = 1220$ (\$). Inspection of Table 3 indicates some high-cost cells were assigned and some low-cost cells bypassed by using the northwest-corer method. Indeed, this is to be expected since this method ignores costs in favor of following an easily programmable allocation algorithm. Table 4 shows a least-cost assignment. (Cell Dummy-E was assigned first, C-E second, B-H third, A-H fourth, and so on.) Total cost: $30 \cdot 3 + 25 \cdot 6 + 15 \cdot 5 + 10 \cdot 10 + 10 \cdot 9 + 20 \cdot 6 + 40 \cdot 12 + 0 \cdot 1 = 1105$ (\$). Table 5 shows the VAM assignments. (Cell Dummy-G was assigned first, B-F second, C-E third, A-H fourth, and so on.) Note that this starting solution is very close to the optimal solution obtained after making all possible improvements (see next chapter) to the starting solution obtained using the northwest-corer method. (See Table 3) Total cost: $15 \cdot 14 + 15 \cdot 10 + 10 \cdot 10 + 20 \cdot 4 + 20 \cdot 1 + 40 \cdot 5 + 35 \cdot 7 + 0 \cdot 1 = 1005$ (\$).



From	To	E	F	G	H	Factory Supply	
A		10	4	30	25	15	14
B		20	10	15	20	10	10
C		10	1	30	20	20	15
D		30	40	35	12	45	12
Dummy		0	0	0	1	0	1
Destination Requirements		10	15	12	15	52	52

Table 3 "Northwest – Corner Assignment"

From	To	E	F	G	H	Factory Supply	
A		10	3	30	25	15	14
B		20	15	6	20	10	10
C		9	10	30	6	20	15
D		30	12	40	35	45	12
Dummy		1	0	0	0	0	1
Destination Requirements		10	15	12	15	52	52

Table 4 "Least - Cost Assignment"

From	To	E	F	G	H	Factory Supply	
A		10		30	25	15	14
B		20	10		20	10	10
C		10		30		20	15
D		30	5	40	4	35	12
Dummy		0	7	0	0	0	1
Destination Requirements		10	15	12	15	52	52

Table 5 "VAM Assignment"

Develop Optimal Solution: To develop an optimal solution in transportation problem involves evaluating each unused cell to determine whether a shift into it is advantageous from a total-cost stand point. If it is, the shift is made, and the process is repeated. When all cells have been evaluated and appropriate shifts made, the problem is solved. One approach to making this evaluation is the Stepping stone method. The term stepping stone appeared in early descriptions of the method, in which unused cells were referred to as "water" and used cells as "stones"— from the analogy of walking on a path of stones half-submerged in water. The stepping stone method was applied to the VAM initial solution, as shown in Table 5. Table 6 shows the optimal solutions reached by the Stepping stone method. Such solution is very close to the solution found using VAM method.

From	To	E	F	G	H	Factory Supply	
A		10		30	25	15	14
B		20	10	15	20	10	10
C		10		30	4	20	15
D		30	4	40	8	35	12
Dummy		0	1	0	0	0	1
Destination Requirements		10	15	12	15	52	52

Table 6 "Optimal Matrix, With Minimum Transportation Cost of \$1,000."

Alternate Optimal Solutions : When the evaluation of any empty cell yields the same cost as the existing allocation, an alternate optimal solution exists (see Stepping Stone Method – alternate solutions). Assume that all other cells are optimally assigned. In such cases, management has additional flexibility and can invoke non-transportation cost factors in deciding on a final shipping schedule.



From	To	E	F	G	H	Factory Supply
A		10	30	25	15	14
B		20	9	15	20	10
C		10	10	30	20	15
D		30	5	40	35	12
Dummy		0	0	0	0	1
Destination Requirements		10	15	12	15	52

Table 7 "Alternate Optimal Matrix for the Chest Transportation Problem, With Minimum Transportation Cost of \$1,000. Degeneracy: Degeneracy exists in a transportation problem when the number of filled cells is less than the number of rows plus the number of columns minus one ($m + n - 1$). Degeneracy may be observed either during the initial allocation when the first entry in a row or column satisfies both the row and column requirements or during the Stepping stone method application, when the added and subtracted values are equal. Degeneracy requires some adjustment in the matrix to evaluate the solution achieved. The form of this adjustment involves inserting some value in an empty cell so a closed path can be developed to evaluate other empty cells. This value may be thought of as an infinitely small amount, having no direct bearing on the cost of the solution. Procedurally, the value (often denoted by the Greek letter epsilon, ϵ) is used in exactly the same manner as a real number except that it may initially be placed in any empty cell, even though row and column requirements have been met by real numbers. A degenerate transportation problem showing a Northwest Corner initial allocation is presented in Table 8, where we can see that if ϵ were not assigned to the matrix, it would be impossible to evaluate several cells. Once a ϵ has been inserted into the solution, it remains there until it is removed by subtraction or until a final solution is reached. While the choice of where to put an ϵ is arbitrary, it saves time if it is placed where it may be used to evaluate as many cells as possible without being shifted.

From	To	D1	D2	D3	Source Supply
S1		20	ϵ	30	20
S2		10	12	30	12
S3		30	3	40	8
Destination Requirements		20	15	5	40

Table 8 "Degenerate Transportation Problem With ϵ Added. Number of filled cells = 4"

Transportation Problem with Maximization as a Criterion: A fictive corporation A has a contract to supply motors for all tractors produced by a fictive corporation B. Corporation B manufactures the tractors at four locations around Central Europe: Prague, Warsaw, Budapest and Vienna. Plans call for the following numbers of tractors to be produced at each location: Prague 9 000, Warsaw 12 000, Budapest 9 000, Corporation A has three plants that can produce the motors. The plants and production capacities are Hamburg 8 000, Munich 7 000, Leipzig 10 000, Dresden 5000. Due to varying production and transportation costs, the profit earns on each motor depends on where they were produced and where they were shipped. The following transportation table (Table 9) gives the accounting department estimates of the euro profit per unit (motor).

Shipped to Produced at	Prague	Warsaw	Budapest	Source Capacity
Hamburg	70	90	130	8 000
Munich	80	130	60	7 000
Leipzig	65	110	100	10 000
Dresden	95	80	35	5 000
Destination Capacity	9 000	12 000	9 000	30 000

Table 9 "The Euro Profit Per One Shipped Motor"



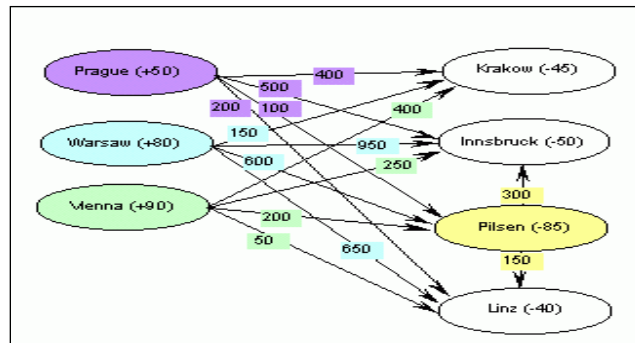
Table 10 shows a highest - profit assignment (Least Cost method modification). In contrast to the Least – Cost method it allocates as much as possible to the highest-cost cell. (Cell Hamburg - Budapest was assigned first, Munich - Warsaw second, Leipzig - Warsaw third, Leipzig – Budapest fourth, Dresden – Prague fifth and Leipzig – Prague sixth.) Total profit : 3 335 000 euro.

Shipped to Produced at	Prague	Warsaw	Budapest	Source Capacity
Hamburg	70	90	130	8 000
Munich	80	130	60	7 000
Leipzig	65	110	100	10 000
Dresden	95	80	35	5 000
Destination Capacity	9 000	12 000	9 000	30 000

Table 10 "Highest - Profit Assignment"

Applying the Stepping Stone method (modified for maximization purposes) to the initial solution we can see that no other transportation schedule can increase the profit and so the Highest – Profit initial allocation is also an optimal solution of this transportation problem.

The Transshipment Problem: The transshipment problem is similar to the transportation problem except that in the transshipment problem it is possible to both ship into and out of the same node (point). For the transportation problem, you can ship only from supply points to demand points. For the transshipment problem, you can ship from one supply point to another or from one demand point to another. Actually, designating nodes as supply points or demand points becomes confusing when you can ship both into and out of a node. You can make the designations clearer if you classify nodes by their net stock position-excess (+), shortage (-), or 0. One reason to consider transshipping is that units can sometimes be shipped into one city at a very low cost and then transshipped to other cities. In some situations, this can be less expensive than direct shipment. Let's consider the balanced transportation problem as an example. Picture 1 shows the net stock positions for the three warehouses and four customers. Say that it is possible to transship through Pilsen to both Innsbruck and Linz. The transportation cost from Pilsen to Innsbruck is 300 euro per unit, so it costs less to ship from Warsaw to Innsbruck by going through Pilsen. The direct cost is 950 euro, and the transshipping cost is 600 + 300 = 900 euro. Because the transportation cost is 300 euro from Pilsen to Innsbruck, the cost of transshipping from Prague through Pilsen to Innsbruck is 400 euro per unit. It is cheaper to transship from Prague through Pilsen than to ship directly from Prague to Innsbruck.



Picture 1 "Transshipment Example in the Form of a Network Model"

There are two possible conversions to a transportation model. In the first conversion, make each excess node a supply point and each shortage node a demand point. Then, find the cheapest method of shipping from surplus nodes to shortage nodes considering all transshipment possibilities. Let's perform the first conversion for the Picture 7.1 example. Because a transportation table Prague, Warsaw, and Vienna have excesses, they are the supply points. Because Krakow, Pilsen, Innsbruck, and Linz have shortages, they are the demand points. The cheapest cost from Warsaw to Innsbruck is 900 euro, transshipping through Pilsen. The cheapest cost from Prague to Innsbruck is 400 euro, transshipping through Pilsen too. The cheapest cost from all other supply points to demand points is obtained through direct shipment. Table 7.11 shows the balanced transportation table for this transshipment problem. For a simple transportation network, finding all of the cheapest routes from excess nodes to shortage nodes is easy. You can list all of the possible routes and select the cheapest. However, for a network with many nodes and arcs, listing all of the possible routes is difficult.



From \ To	Krakow	Innsbruck	Pilsen	Linz	Source Supply
Prague	400	400	100	200	50
Warsaw	150	900	600	650	80
Vienna	400	250	200	50	90
Destination Demands	45	50	85	40	220
					220

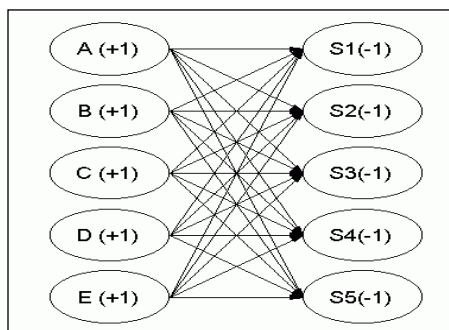
Table 11 "The Transshipment Problem After Conversion to a Transportation Model"

The second conversion of a transshipment problem to a transportation model doesn't require finding all of the cheapest routes from excess nodes table to shortage nodes. The second conversion requires more supply and demand nodes than the first conversion, because the points where you can ship into and out of, occur in the converted transportation problem twice – first as a supply point and second as a demand point.

The Assignment Problem: Another transportation problem is the assignment problem. You can use this problem to assign tasks to people or jobs to machines. You can also use it to award contracts to bidders. Let's describe the assignment problem as assigning n tasks to n people. Each person must do one and only one task, and each task must be done by only one person. You represent each person and each task by a node. Number the people 1 to n, and number the tasks 1 to n. The assignment problem can be described simply using a verbal model or using a linear programming mathematical model. For example, say that five groups of computer users must be trained for five new types of software. Because the users have different computer skill levels, the total cost of trainings depends on the assignments.

Software Types	S1	S2	S3	S4	S5
User Groups					
A	5	4	6	4	1
B	2	5	4	10	5
C	10	12	10	6	8
D	1	3	4	2	6
E	2	5	8	11	7

Table 12 "Cost of Trainings According to the Groups of Users"



Picture 2 "Network Model for Assignment Problem"

Table 12 shows the cost of training for each assignment of a user group (A through E) to a software type (S1 through S5). Picture 2 is a network model of this problem. A balanced assignment problem has the same number of people and tasks. For a balanced assignment problem, the relationships are all equal. Each person must do a task. For an unbalanced assignment problem with more people than tasks, some people don't have to do a task and the first class of constraints is of the \leq type. In general, the simplex method does not guarantee that the optimal values of the decision variables are integers. Fortunately, for the assignment model, all of the corner point solutions have integer values for all of the variables. Therefore, when the simplex method determines the optimal corner point, all of the variable values are integers and the constraints require that the integers be either 1 or 0 (Boolean).



Conversion to a Balanced Transportation Table

It's not surprising that the variable values for corner point solutions to the assignment model are integers. The assignment model is a special case of the transportation problem, and the transportation problem has integer variable values for every corner point. For the assignment model, the number of supply and demand points are both n. The supply points correspond to each person, and the demand points correspond to each task. Furthermore, every supply amount is 1 and every demand amount is 1. There is one of each person and one of each task.

Software Types	S1	S2	S3	S4	S5	Source Supply
User Groups						
A	5	4	6	4	1	1
B	2	5	4	10	5	1
C	10	12	10	6	8	1
D	1	3	4	2	6	1
E	2	5	8	11	7	1
Destination Demands	1	1	1	1	1	5

Table 13 "The Computer Users Assignment Problem in the Transportation Table Format"

Table 13 represents the computer user's assignment problem in the balanced transportation table format. For the computer users assignment problem, you minimize the total cost of training. Because number of users = 5 and software types = 5, the transportation problem is balanced. You can use standard method for initial solution finding (Northwest-Corner method, Least-Cost method, or Vogel's approximation method (VAM) and a Stepping-Stone method for developing an optimal solution. Thinking of the transportation problem as shipping items between cities is helpful, but it's the mathematical structure that is important. The assignment problem has the same mathematical structure as the transportation problem, even though it has nothing to do with shipping units. Note, that each feasible solution of assignment problem will always be strongly degenerated (number of nonzero variables will always be n).

CONCLUSION: The transportation problem is only a special topic of the linear programming problems. It would be a rare instance when a linear programming problem would actually be solved by hand. There are too many computers around and too many LP software programs to justify spending time for manual solution.

REFERENCES:

- Hammer P L. Time minimizing transportation problem. *Nav. Res. Log. Quart.*, 1969, 16(3): 345–357.
- Hammer P L. Time minimizing transportation problem. *Nav. Res. Log. Quart.*, 1971, 18: 487–490.
- Garfinkel R S, Rao M R. The bottleneck transportation problem. *Nav. Res. Log. Quart.*, 1971, 18: 465–472.
- Szware W. Some remarks on the time transportation problem. *Nav. Res. Log. Quart.*, 1971, 18: 473–485.
- Bhatia H L, Swarup K, Puri M C. Time-Cost trade off in a transportation problem *Opsearch.* 1976, 13(3-4): 129–142.
- Ramakrishnan C S. A note on the time minimizing transportation problem. *Opsearch*, 1977, 14: 207–209.
- Sharma J K, Swarup K. Time minimizing transportation problems. *Proceeding of Indian Academy of Science (Mathematical Science)*, 1977, 513–518.
- Sessan C R, Tikekar V G. On Sharma-Swarup algorithm for time minimizing transportation problems. *Proceeding of Indian Academy of Science (Mathematical Science)*, 1980, 101–102.
- Prakash S. Transportation problem with objectives to minimize total cost and duration of transportation. *Opsearch*, 1981, 184(4): 235–238.
- Bhatia H L, Swarup K, Puri M C. A procedure for time minimization transportation problem. *Indian Journal of Pure Applied Mathematics*, 1977, 8(8): 920–929.
- Shiang T L. The total cost bounds of the transportation problem with varying demand and supply. *Omega*, 2003, 31(4): 247–251.
- Carlsson C, Korhonen P. A parametric approach to fuzzy linear programming. *Fuzzy Sets and Systems*, 1986, 20(1): 17–30.
- Chanas S. The use of parametric programming in fuzzy linear programming. *Fuzzy Sets and Systems*, 1983, 11(2): 243–251.
- Ahmad, H.A., 2012. The best candidate's method for solving optimization problems. *J. Comput. Sci.*, 8: 711-715. DOI: 10.3844/jcssp.2012.711.715