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Study about Topology, Base and Subbase for a Topology

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Abstract : The word topology is derived from two Greek words, topos meaning surface and logs meaning discourse or study. Topology thus literally means the study of surfaces or the science of position. The subject of topology can now be defined as the study of all topological properties of topological spaces. A topological property is a property which if possessed by a topological space X, is also possessed by every homeomorphic image of X. If very roughly, we think of a topological space as a general type of geometric configuration, say, a diagram drawn on a sheet of rubber, then a homemorphism may be thought of as any deformation of this diagram (by stretching bending etc.) which does not tear the sheet. A circle can be deformed in this way into an ellipse, a triangle, or a square but not into a figure eight, a horse shoe or a single point. Thus a topological property would then be any property of the diagram which is invariant under (or unchanged by) such a deformation. Distances, angles and the like are not topological properties because they can be altered by suitable non-tearing deformations. Due to these reasons, topology is often described to non-mathematicians as "rubber sheet geometry".

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Maurice Frechet (1878-1973) was the first to extend topological considerations beyond Euclidean spaces. He introduced metric spaces in 1906 in a context that permitted one to consider abstract objects and not just real numbers or n-tupls of real numbers. Topology emerged as a coherent discipline in 1914 when Felix Hausdorff (1868-1942) published his classic treatise Grundzuge der Mengenlehre. Hausdorff defined a topological space in terms of neighbourhoods of member sof a set.

Base and Subbase for a Topology

A topology on a set can be complicated collection of subsets of a set and it can be difficult to describe the entire collection. In most cases one describes a subcollection that generates the topology. One such collection is called a basis and another is called a subbasis.

Definition. A sub family B of T is called a base for the topology T on X iff for each point x of the space and each nbd U of x, There is a member V of B such that $x \in V \subset U$

For example, in a metric space every open set can be expressed as a union of open balls and consequently the family of all open balls is a base for the topology induced by the metric.

The following is a simple characterization of basis and is frequently used as a definition.

Definition. A subfamily B of a topology T is a base for T if and only if each member of T is the union of members of B.

To prove that this second definition is equivalent to first one, suppose that B is a base for the topology T and that $U \in T$.

Let V be the union of all members of B which are subsets of U and suppose that $x \in U$. Then there is W in B such that $x \in W \subset U$ and consequently $x \in V$ and since V is surely a subset of U,

V = U. So the first definition _ second.

Conversely let B be a subfamily of T and each member of T is the union of members of B. If $U \in T$, then U is the union of members of subfamily B and for each x in U, there is a V in B such that $x \in V \subset U$. Consequently B is a base for T.

Example.

(1) The collection B of all open intervals is a basis for the usual topology on R.

(2) The collection B of all open disks is a basis for the usual topology on the plane

(3) If X is a set, then B = {{x} [$x \in X$} is a basis for the discrete topology on X.

(4) Let (X, d) be a metric space, then the family B = {B (x, ∈) ; $x \in X$ and ∈ > 0} is a basis for the topology generated by d.

TOPOLOGICAL SPACES

The ideas of metric and metric spaces are abstractions of the concept of distance in Euclidean space. These abstractions are fundamental and useful in all branches of mathematics.

Definition. A metric on a set X is a function $d : X \times X \rightarrow R$ that satisfies the following conditions.

(a) $d(x, y) \geq 0$ for all $x, y \in X$
(b) $d(x, y) = 0$ if and only if $x = y$
(c) $d(x, y) = d(y, x)$ for all $x, y \in X$.
(d) $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$.
If d is a metric on a set X, ordered pair (X, d) is called a metric space and if $x, y \in X$,
Then d(x, y) is the distance from x to y.

Metric Space

Note that a metric space is simply a set together with a distance function on the set.
The function $d : R \times R \rightarrow R$ defined by $d(x, y) = |x-y|$ satisfies the four conditions of the definition and hence this function is a metric on R. It is called the usual metric on R. Also the function d :
R2 $\times$ R2$\rightarrow$R defined by d{(x1 x2), (y1, y2)} = 2
(x1 – x2) + (y – y) is called the usual metric on R2.

Metrizable

A metrizable space is a topological space X with the property that there exists at least one metric on the set X whose class of generated open sets is precisely the given topology i.e. it is a topological space whose topology is generated by some metric. But metric space is a set with a metric on it. The following example shows that there are topological spaces that are not metrizable.

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