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Exploring Graph Theory For Practical Solutions

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Abstract
Graph theory is widely used to prove many mathematical theorems and models. This paper present the various applications and techniques of graph theory to solve problems in different fields of science and technology in addition to mathematics. A graph can be used to represent almost any physical situation involving discrete objects and a relationship among them. This abstract provides a concise overview of graph theory's foundational principles, including graph types(such as directed, undirected, weighted, and unweighted graphs), basics terminologies(vertices, edges, paths, cycles), and essential theorems (e.g, Euler's theorem Hamiltonian cycles). Moreover, it highlights practical applications of graph theory, such as shortest path algorithms (e.g., Dijkstra's algorithm), network flow optimization, and graph coloring problems. By unraveling the intricacies of graph theory, this abstract aims to foster a deeper understanding of its role in shaping modern computational paradigms and problem-solving methodologies. These fields include website design, chemistry, biology, computer science, software engineering & operations research.
Keywords:- Eulerian graph, Hamiltonian graphs and cycles, Chromatic number, Tree, Kuratowski's theorem, Dijkstra's algorithm.

Definition of a graph
A graph is a collection of vertices and edges. Vertices can be thought of as dots that are connected by edges.For the purpose of this paper, we will assume that the graph has at most one edge between any two vertices.Graphs can be very resourceful tools used in real life in order to help people see where they can go and the different routes they can take to get there.[2].In graph theory, graphs are used to model relation-ships between objects, with vertices representing entities and edges representing the connections or interactions between them. The study of graph theory involves exploring various properties and characteristics of graphs, such as connectivity, paths, cycles, and coloring. It also investigated different types of graphs, including directed graphs (where edges have associated weights pr costs). Graph theory provides a powerful framework for analyzing and solving problems in diverse areas, including computer science, operations research, biology, telecommunication, and social sciences. There are various examples of these graphs, but for now we will use Konigsberg Bridge Problem as an example. This problem consists of two islands in which seven bridges connected them and other various islands. The problem states that we can walk through the edges only onceand we have to end up at the same place we started. However, we

realized that it is impossible to go through every bridge exactly once and end up where we started. The Konigsberg Bridge problem can be represented in a graph where the edges can be trans-versed either way, making it an undirected graph (as shown in fig.1). An undirected graph is a graph that does not contain any arrows on its edges, indicating which way to go. A directed graph, on the other hand, is a graph in which its edges contain arrows indicating which way to go.

Figure 1: Konigsberg Bridge

Aim and Objectives of Study: This paper aims to foster a deeper understanding of its role in shaping modern computational paradigms and problem-solving methodologies of various graphs and algorithms.

Preliminaries

Graph- A graph is denoted as G(V,E) graph consisting of two set vertices 'V' and edges 'E'. In Mathematics, a graph is a pictorial representation of any data in an organised manner. The graph shows the relationship between
variable quantities. In a graph theory, the graph represents the set of objects, that are related in some sense to each other. The objects are basically mathematical concepts, expressed by vertices or nodes and the relation
between the pair of nodes, are expressed by edge.

Basic Terminology or Types of graphs

1. Directed Graph : A graph consist the direction of edges then this is called di directed graph.As shown in fig.2[1,2]

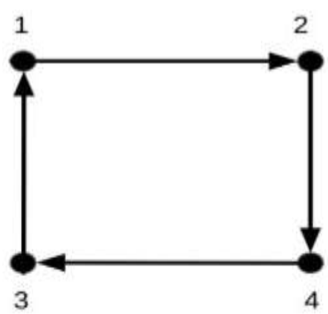


Figure 2: A Direct graph [Ref 2]

2. Undirected Graph: A graph which is not directed then it is called undirected graph.As shown in fig.3.[1,2]

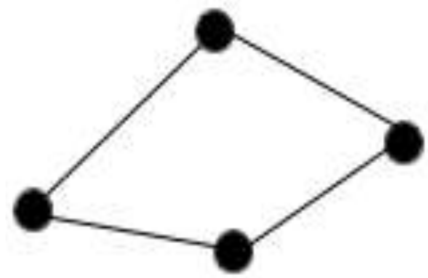


Figure 3: An Undirected Graph [Ref 2]

3. Complete Graph: A simple connected graph is said to be complete if each ver-tex is connected to every other vertex.[1].

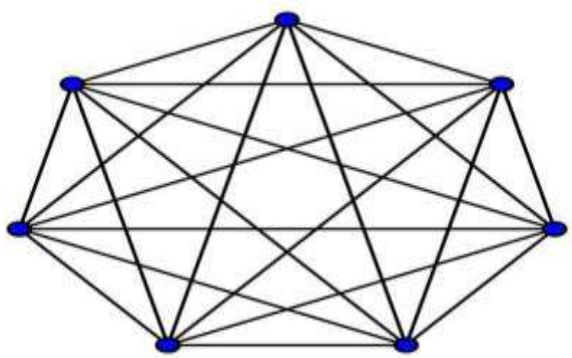


Figure 4: Complete Graph [1]

4. Bigraph or Bipartite): If the vertex set V of a graph G can be partitioned into two non- empty disjoint subsets X and Y in such a way that edge of G has one end in X and one end in Y. Then G is calledbipartite.[1].

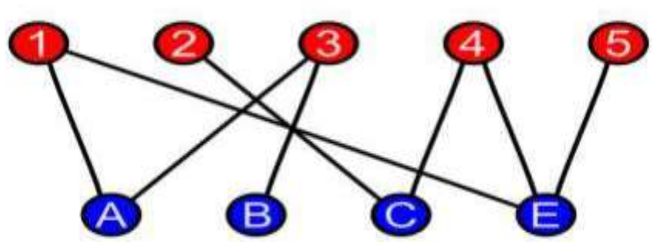


Figure 5: Bigraph or Bipartite [1]

5. CONNECTED GRAPH: An undirected graph is said to be connected if there is a path between every two vertices.[1].

Remarks- : If a graph is connected then it will not bipartite

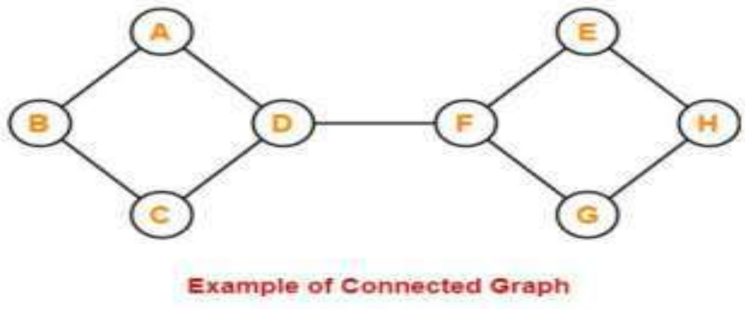


Figure 6: Connected Graph [1]

3. Eulerian path: A path in a graph is said to be an eulerian path if it traverses each edge in the graph once and only once.

There is a simple formula relating the numbers of vertices, edges and faces in a connected plane graph. It is known as Euler's formula because Euler established it for those plane graphs defined by the vertices and edges of polyhedra. [1,3]

Theorem 3.1.: If G is a connected plane graph, then v- ε+ Φ =2

Proof By induction on **Φ**, the number of faces of G. If Φ = 1, then each edge of G is a cut edge and so G, being connected, is a tree. In this case **ε** = v -1, Suppose that it is true for all connected plane graphs with fewer than n faces, and let G be a connected plane graph with n ≥ 2 faces. 'Choose an edge e of G that is not a cut edge. Then G - e is a connected plane graph and has n - 1 faces, since the two faces of G separated by e combine to form one face of G - e. By the induction hypothesis.[3]

$$v(G - e) - B\,(G- e) + \Phi\,(G - e) = 2$$

and, using the relations.

$v(G - e) = v(G)$ $\varepsilon\,(G - e) = \varepsilon\,(G)-l$ $\Phi\,(G - e) = \Phi\,(G)-1$

we obtain,

$$v(G)- \varepsilon\,(G)+ \Phi\,(G)=2$$

The theorem follows by the principle of induction.[3].

4.Hamiltonian graph: A connected graph which contain Hamiltonian circuit is called Hamiltonian graph.[1]

Hamiltonian circuit: A circuit that passes through each of the vertices in a group G exactly one except the starting vertex and end
vertex is called Hamiltonian circuit[3]..

A path that contains every vertex of G is called a Hamilton path of G; similarly, a Hamilton cycle of G is a cycle that contains every vertex of G. Such paths and cycles are named after Hamilton (1856), who described, in a letter to his friend Graves, a mathematical game on the dodecahedron '(figure .7a) in which one person sticks five pins in any five consecutive vertices and the other is required to complete the path so formed to a spanning cycle.[3]

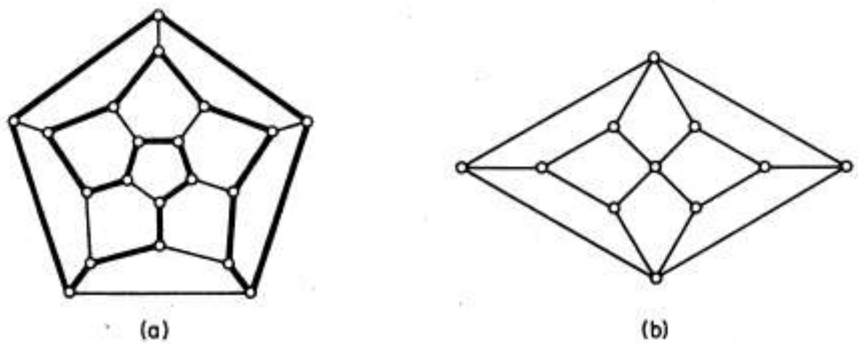


Figure 7a: *(a)* The dodecahedron; *(b)* the Herschel graph.[3]

A graph is *hamiltonian* if it contains a Hamilton cycle. The dodecahedron is hamiltonian (see figure *7a);* the Herschel graph (figure *7b)* is non hamiltonian, because it is bipartite and has an odd number of vertices.

Theorem **5.1** A s~mple graph is hamiltonian if and only if its closure is **hamiltonian.[3].**

Lemma **5.1.1.** Let G be a simple graph and let *u* and *v* be nonadjacent vertices in G such that d(u)+d(v) ≥ (5.1)

Then G is hamiltonian if and only if G + *uv* is hamiltonian.

Proof Apply lemma 5.1.1 each time an edge is added in the formation of the closure

Theorem 5.1: Has a number· of interesting consequences. First, upon making the trivial observation that all complete graphs on at least three vertices are hamiltonian, we obtain the following result.

Corollary **5.1.** Let G be a· simple graph with *v* >3. If c~(G) is complete,then G is. hamiltonian.

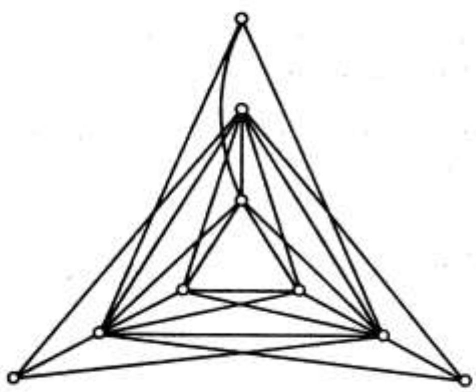


Figure 7c: **A hamiltonian graph** [3].

Consider, for example, the graph of figure 7c. One readily checks that its closure is complete. Therefore, by corollary 5.1, it is hamiltonian. It is perhaps interesting to note that the graph of figure 7c, can be obtained from the graph of figure 7.c by altering just one end of one edge, and yet we have results (corollary 5.1 and theorem 5.1) which tell us· that· this one is hamiltonian .[3].

Applications Hamiltonian cycles and Hamiltonian paths: while challenging to find in large graphs, have numerous practical applications across various fields. Some of the key applications include:

1. Transportation and Logistics: In transportation networks, finding Hamiltonian cycles can optimize delivery routes for goods and services, minimizing travel time and costs. These cycles ensure that each location is visited exactly once, leading to efficient resource utilization and improved logistics [7,8].
2. Computer Networks: Hamiltonian paths play a crucial role in designing optimal data transmission routes between nodes in computer networks. By determining the shortest path that visits each node exactly once, network latency can be reduced, resulting in faster data transfer and enhanced network performance [9,10].
3. Circuit Design and Chip Testing: Hamiltonian cycles and paths are utilized in circuit design and chip testing to verify the correctness of connections and detect faults in integrated circuits. Ensuring the presence of a Hamiltonian cycle in a circuit guarantees the reliability and proper functioning of electronic devices. [11,12].
4. Bioinformatics and Molecular Chemistry: In bioinformatics, Hamiltonian cycles and paths are applied to model protein folding patterns, DNA sequencing, and drug discovery. Identifying Hamiltonian paths in protein structures aids in understanding their functions and interactions, contributing to drug development and disease research [13,14].
5. Robotics and Path Planning: In robotics, Hamiltonian paths are employed to plan efficient and collision-free paths for robots navigating through complex environments. The path should cover all necessary locations without revisiting any vertex, ensuring that the robot reaches its destination optimally [13].

6. EXAMPLE ON ALGORITHMS GRAPHS

6.1. Dijkstra Algorithm

6.1.1. The shortest path problem

The Dijkstra Algorithm is used to find the shortest path from a 'source node' to other nodes in the graph. A node is a place, person, or object and the thing that connects these nodes are the edges. The example we'll be using to demonstrate this today is the example the inventor himself, Dijkstra, used. This problem includes a weighted graph which is a graph that has some type of weight or cost on the edges of the graph, a weight being the length of a path. These weights or costs can be anything from distance or time, to anything that represents a connection between the nodes. Once these nodes have been visited, meaning that one has seen the path to take from the source node, which is going to be node 1 for this section, to another node, one can put a red mark next to the number to signal that. In this example there is a graph with four nodes and four weighted edges. We are trying to see what the shortest path is from node 1 to the other nodes which can be the distance, time, or anything that models the" connection" between the pair of nodes connected.

6.2. Dijkstra Algorithm Steps

To solve the problem above the first step is to see what the shortest path is from the source node to node 1, which are the same thing. Since the distance from the source node to the rest have not yet been established, we will put infinity signs next to the nodes that are not visited. Once nodes have been visited and the shortest path has been discovered we can add them to the path. Because we are starting from[2].

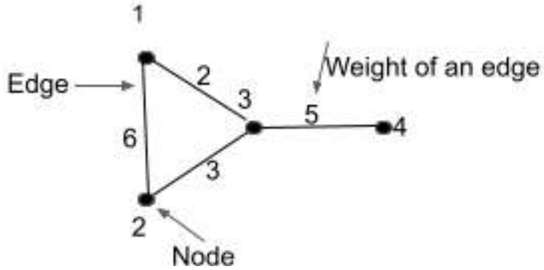
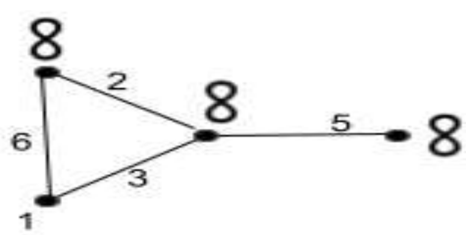


Figure 8: An image of a weighted graph [2].
node 1 as our source node we can mark it as visited and included in the path. Then we check the length of the path of the nodes adjacent to the source node and write down the distance from the source node to its adjacent nodes. Some adjacent nodes are 1 and 2 with a weighted edge of 6. Because we have discovered the distance and the shortest way to reach node 2 from the source node, we can put 6 on node 2. Then we look at the node adjacent to node 2 and repeat the same steps as before. When finding the distance between two edges connecting two nodes one must add the weight/cost to find the distance between the source node and the adjacent nodes. If the distance from the source node to an adjacent node can be reached in different ways we will check to find the shortest path. If there are different results, we will update it to the shortest path but if there are no differing results, we will not update it.

Figure 9: An image of a weighted graph with no shortest path's discovered.[2]



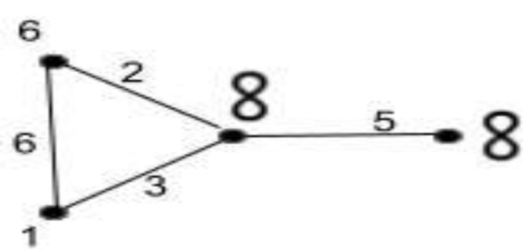


Figure 10: An image of a weighted graph with one shortest path discovered.[2]

6.3 Application of Dijkstra Algorithm

1.It is very popular in the **Geographical Maps**. It is used to locate the points on the map that correspond to the graph's vertices.
2. In order to identify the Open Shortest Path First, it is needed in IP routing.
3. The telephone network makes use of it.
4. Finding the shortest paths between nodes in a weighted graph, which may represent, for example, a road network.

7. Application of graph theory

Graph theory, a branch of discrete mathematics, has numerous application in vari-ous fields.[1,2,3]
1.Computer Network –

Graphs are used to represent networks of communication, Data organisation, computational devices, the flow of computation etc. One practical example is the link structure of a website could be represented by a directed graph

2.Networks : Graph theory is extensively used in the study of networks, such as social Net-works, transportation networks ,communication networks and computer It helps in analysing connectivity, identifying critical nodes, optimising routes, And understanding networks resilience.

3. Computer science: Graph theory plays a vital role in com-puter science, particularly in the design and Analysis of algorithms. It is used in data structures Like adjacent lists for representing graphs. Algorithms like Dijk-stra's algorithm for shortest Paths, Prim's algorithm for minimum Spanning trees, and algorithms for graph Traversal (e.g.,depth-first search and breadthFirst search) are all based on graph theory.[3]

4. Circuit Design: In electrical engineering, graph theory is ap-plied to analyze and Design circuits.Graph models help in representing connections between Components and analysing properties like voltage distribution, current flow and circuit efficiency

5. Operations Research : Graphs are used to model optimization problems like the traveling salesman problem, where the objective is to find the shortest routes that visits a set of cities exactly once and returns to the origin City.

6.Biology: Graph theory is applied in bioinformatics for modelling molecular structures, proteinprotein interactions, and genetics networks.

7. Chemistry : Graph theory is used to model and analyze molecular structures, chemical reac-tions, and chemical bonding
patterns.

8.Transportation : Graph theory is used to model transportation networks and analyze traffic flow, optimising routes, and
minimising congestion.

9. Operation Management :Graph theory is applied in project management to model project activities, de-pendencies, and critical paths.

10. Epidemiology : Graph theory plays a crucial role in modelling the spread of diseases. Nodes represent individuals, and edges represent contacts or interactions between them. Epidemiologists use graph algorithms to simulate dis-ease transmission, identify key influencers, and develop strategies for diseases control.

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15.Power Grids: Graph theory is employed in modelling and analyzing electrical power grids. Nodes represent power stations,

substations, and consumers, while edges represent transmission lines. Graph algorithms help in optimizing power flow identifying vul-nerabilities, and designing resilient power grid systems.
16.Game Theory: Graph theory is applied in graph theory to model strategic interactions between players in various games,including social dilemmas, voting systems, and economic competitions. Graph algorithms help analyzeequilibrium strategies, coalition formations, and game dynamics.
19. Genetics: Graph theory is used in genetics for analyzing genetic networks, genome sequencing, and evolutionary relationship between species. Graph algorithms help in notifying genetic patterns, predicting gene functions, and understanding genetic diseases.

8. Conclusion

Graph theory plays a fundamental role in discrete mathematics, offering powerful tools and concepts for analyzing and solving a wide range of problems. Through its study, mathematics and researchers gain insights into the structure, connectivity, and properties of networks and discrete structures. The applications if graph theory extend across numerous fields, including computer science, biology, telecommunications, geography, and social sciences, making it a versatile and indispensable area of study. With its rich theoretical foundations and practical applications, graph theory continues to inspire new discoveries and innovations, shaping our understanding of complex systems and networks in the digital age and beyond.

9. Reference

1.Review of Graph Theory and It's Application. *Gyanvendra Pratap Singh and Simran Gupta.* Department of Mathematics and Statistics,Deen Dayal Upadhyaya Gorakhpur University Gorakhpur-273009, (U.P.), India, 2024 JETIR May 2024, Volume 11, Issue 5.
2. Graph Theory and Its Applications**.** *Crystal Egbunike and Wintana Tewolde* . Int. Res. Journal of Science & Engineering, 2020; Special Issue A7: 533-538 SJIF Impact Factor 6.68 ISSN: 2322-0015.
3. A BOOK ON GRAPH THEORY WITH APPLICATIONS,
J. A. Bondy and U. S. R. Murty
Depart,nent· of *Combinatorics and Optimization, University* of *Waterloo, Ontario, Canada'. First published in Great Britain 1976 by 'The· Macmillan Press Ltd'*
4. Advances in Mathematics: Scientific Journal 10 (2021), no.3, 1407–1412 ISSN: 1857-8365
(printed); 1857-8438 (electronic) https://doi.org/10.37418/amsj.10.3.29 (How to write related work)
5. Rishi Pal Sing, Vandana, " Application of Graph Theory in Computer Science and Engineering," International Journal of Computer Applications (0975 – 8887) Volume 104 – No.1, October 2014.
6. S.G.Shirinivas, S.Vetrivel, Dr. N.M.Elango, "Applications Of Graph Theory In Computer Science An Overview," International Journal of engineering Science and Technology Vol. 2(9), 2010, 4610-4621
7. *Bramel, Julien, and David Simchi-Levi.* "The logic of logistics: theory, algorithms, and applications for logistics management." (1997).
8. *Çakir, Esra, Ziya Ulukan, and Tankut Acarman*. "Shortest fuzzy hamiltonian cycle on transportation network using minimum vertex degree and time-dependent dijkstra's algorithm." IFAC-PapersOnLine 54, no. 2 (2021): 348-353.
9. *Chen, Shao Dong, Hong Shen, and Rodney Topor.* "An efficient algorithm for constructing Hamiltonian paths in meshes." Parallel Computing 28, no. 9 (2002): 1293-1305.

10. *Bae, Yongeun, Chunkyun Youn, and Ilyong Chung.* "Application of the Hamiltonian Circuit Latin square to the parallel routing algorithm on 2-circulant networks." In Computational and Information Science: First International Symposium, CIS 2004, Shanghai, China, December 16-18, 2004. Proceedings 1, pp. 219-224. Springer Berlin Heidelberg, 2005.
11. *Leite, Jônatas Boás, and José Roberto Sanches Mantovani.* "Distribution system state estimation using the Hamiltonian cycle theory." IEEE Transactions on Smart Grid 7, no. 1 (2015): 366-375.
12. *Girard, Patrick, Christian Landrault, Serge Pravossoudovitch, and Daniel Severac.* "Reducing power consumption during test application by test vector ordering." In 1998 IEEE International Symposium on Circuits and Systems (ISCAS), vol. 2, pp. 296-299. IEEE, 1998.
13. *Nedjatia, Arman, and Béla Vizvárib.* "Robot path planning by traveling salesman problem with circle neighborhood: Modeling, algorithm, and applications." arXiv preprint arXiv:2003.06712 (2020).
14. *Suman Deswal,Anita Singhrova,* "Application Of Graph Theory In Communication Networks," International Journal of Application or Innovation in Engineering & Management (IJAIEM), Volume 1, Issue 2, October 2012.
15. Graph theory application in developing software test strategies for networking system by *Vladimir. V. Riabov (2007)*
16. Applications of Graph Coloring in Modern Computer science by *Shamim Ahmed* (ijcit, issn 2078-5828 (print), issn 2218-5224 (online), volume 03, issue 02, manuscript code: 130101(2012).
17. Graph Theory and Its Applications
By Jonathan L. Gross, Jay Yellen, Mark Anderson. 3rd Edition
First Published on 2018, eBook Published 5 November 2018, Pub. LocationNew York.
18. Improvement and Application of Dijkstra Algorithms, ISSN 2616-5775 Vol. 5, Issue 5: 97-102, DOI: 10.25236/AJCIS.2022.050513.