



## Linear Algebra and theorems : A Review

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### Abstract

Linear algebra owes its origins to the use of vectors in Cartesian two- and three-dimensional space. Vectors are described as a kind of line segment that is governed by both magnitude and direction. The first real vector space is formed by using vectors to represent physical components such as forces and then adding and multiplying them with scalars. Nowadays, it is feasible to learn linear algebra in any number of dimensions. It is a vector space of size  $n$ . Most of the useful results from 2- and 3-space may be expanded in these higher-dimensional spaces. Because they are invisible to the human eye, the  $n$ -space vectors and  $n$ -tuples are useful for describing data. Using vectors as  $n$ -tuples (ordered lists of  $n$  components), it is possible to summarise and manage data efficiently. 8-dimensional vectors or 8-tuples may be used to represent the Gross National Product of eight countries in economics.

**Key words:** Linear Algebra, Matrix, Vectors, Linear Equation etc

### Introduction

Many students find linear algebra to be a vital subject, for at least two reasons. Only a few mathematical subjects can boast such wide-ranging applications in other branches of mathematics (multivariate calculus; differential equations; probability) as well as other branches of physics (biology; chemistry; economics and finance; psychology and sociology) and all engineering fields. As a second benefit, sophomores have a great lot of opportunity to enhance their capacity to cope with abstract concepts. With its vast theoretical foundation and many practical applications in a wide range of scientific and technological disciplines, linear algebra is a well-known branch of mathematics. Many years of study have gone into solving systems of linear equations and determining determinants, just to name a few. In this place, the development of linear algebra and matrix theory gets started. During the early stages of digital computer development, a lot of attention was given to the matrix calculus. Alan Turing and John von Neumann are



two of the most prominent pioneers in computer science. As a consequence of their efforts, computer linear algebra has substantially improved.

### **Linear Algebra**

“The study of vectors, vector spaces (also known as linear spaces), linear mappings (also known as linear transformations), and systems of linear equations falls under the umbrella of linear algebra. Linear algebra is utilised extensively in both abstract algebra and functional analysis because vector spaces constitute a key issue in contemporary mathematics. In addition to its abstract form in linear algebra, analytic geometry and operator theory also generalise it. Nonlinear models may frequently be approximated by linear ones, which has a wide range of applications in both the natural and social sciences.

### **Some Useful Theorems**

- Every vector space has a basis.
- To put it another way, the dimension of a given vector space is well-defined since any two bases in that vector space have the exact same cardinality.
- If and only if the determinant of a matrix is nonzero, it is invertible.
- There must be an isomorphism between two linear maps to be an invertible Matrix.
- It is invertible if the square matrix has either the left or right inverse (see invertible matrix for other equivalent statements).
- If and only if each of its eigenvalues is higher than or equal to zero, a matrix is considered positive semidefinite.
- If and only if each of its eigen values is larger than zero, a matrix is positive definite.
- If and only if a  $n \times n$  matrix has  $n$  linearly independent eigenvectors, it is diagonalizable (i.e. there exists an invertible matrix  $P$  and a diagonal matrix  $D$  such that  $A = PDP^{-1}$ ).
- An orthogonal diagonalizability theorem asserts that a symmetric matrix can only be orthogonally diagonalizable.

### **Linear Equation**

There are two ways to solve a linear equation: you may use either a constant or a constant multiplied by one variable. One or more variables may be included in a linear equation.



As a matter of fact, there are many linear equations in almost every branch of mathematics and in applied mathematics. Many nonlinear equations may be reduced to linear equations by assuming that parameters of interest fluctuate only little from a background condition, and hence they naturally emerge when describing many processes. Exponents are not included in linear equations. The genuine solutions of a single problem are examined in this article. For complicated solutions and, more broadly, linear equations with coefficients and solutions in any discipline, all of its material is applicable.

### **Systems of Linear Equations:**

Breakeven and equilibrium points can only be found by fully comprehending two linear equations at once. For real-world applications, we'll look at examples where linear mathematical equations with two or more variables must be solved.

In real life, the applications of linear equations are vast. To tackle real-life problems using algebra, we convert the given situation into mathematical statements in such a way that it clearly illustrates the relationship between the unknowns (variables) and the information provided.

#### **Applications of Linear equations in Real life**

- Finding unknown age
- Finding unknown angles in geometry
- For calculation of speed, distance or time
- Problems based on force and pressure

### **Modeling with an Algebraic Expression**

Algebraic expressions are the idea of expressing numbers using letters or alphabets without specifying their actual values. The basics of algebra taught us how to express an unknown value using letters such as x, y, z, etc. These letters are called here as variables. An algebraic expression can be a combination of both variables and constants. Any value that is placed before and multiplied by a variable is a coefficient. Using numbers, variables, and operations, an algebraic model is a mathematical assertion. Algebraic



expressions and algebraic phrases are examples of mathematical statements. To represent a situation using an algebraic model:

- Use the actions that offer operations to connect all the pieces of the issue.
- Set up some variables to represent the unknowable (s).
- Using the variables and operations, create an algebraic model of the activities.

### Algebraic Models

Algebraic equations may be used to explain certain processes, either directly or implicitly as a differential equation's solution. An equation is generally defined by using some physics rule like conservation of mass or conservation of momentum or a time/space dependent equation that describes the movement of anything in time and space. An example of an explicit algebraic model is shown in the figure:

$$\text{age} = x - \text{date of birth},$$

where  $x$  is the date of this post. It is possible to think about drug binding to a receptor in terms of an implicit algebraic equation. For a better understanding of the dynamics, consider a differential equation with a straightforward algebraic solution, which relates changes in the proportion of bound receptors with rate differences between receptor formation and information.

$$b = 1 - e^{-(kD+i)t},$$

where  $b$  is the percentage of receptors that are bound,  $kD$  is the rate at which receptors are made bound,  $i$  is the rate at which receptors are unmade bound, and  $t$  is the amount of time.

A simple series of values for the independent variable and the corresponding values of the model's dependent variable is generally all that is required to investigate algebraic models”.



## Conclusions

Linear algebra is a main important part of the mathematics. It is a principal branch of mathematics that is related to mathematical structures closed under the operations of addition and scalar multiplication and that includes the theory of systems of linear equations, matrices, determinants, vector spaces, and linear transformations. Linear algebra, is a mathematical discipline that deals with vectors and matrices and, more generally, with vector spaces and linear transformations. Unlike other parts of mathematics that are frequently invigorated by new ideas and unsolved problems, linear algebra is very well understood. Its value lies in its many applications, from mathematical physics to modern algebra and its usage in the engineering and medical fields such as image processing and analysis.

Overall, in addition to its mathematical usages, linear algebra has broad usages and applications in most of engineering, medical, and biological field. As science and engineering disciplines grow so the use of mathematics grows as new mathematical problems are encountered and new mathematical skills are required. In this respect, linear algebra has been particularly responsive to computer science as linear algebra plays a significant role in many important computer science undertakings.

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